Hydraulics of Steady Flow in Open Channels

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BY

SHERMAN M. WOODWARD

Consulting Planning Engineer,
Tennessee Valley Authority, formerly
Professor at the State University of Iowa

AND

CHESLEY J. POSEY

Professor of Hydraulics and Structural Engineering,
State University of Iowa

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PREFACE

The purpose of this book is to present the theory of the steady flow of water in open channels in concise form, suitable for use in senior and first-year graduate courses, and for home study by young engineers who wish to improve their knowledge of this branch of hydraulics. It is hoped that the book will also prove useful to practicing engineers who have to make computations involving open-channel flow. The tables are unusually complete and up to date, and are so arranged that they can be used with a minimum of reference to the text. A knowledge of elementary hydraulics is presupposed.

Much of the basic subject matter appears in a form nearly identical with that in which it appeared in the Technical Reports of the Miami Conservancy District, under authorship of the senior author. Thanks are due the District for permission to use this material, for which no adequate substitute has been found. The uniform-flow tables and the table of Bresse's function are taken from the open-channel tables of the Iowa Institute of Hydraulic Research, which were computed under the direction of the junior author. Thanks are due the Institute for permitting the inclusion of these tables.

A number of different methods of computing backwater curves are described, each possessing marked advantages in application to certain types of problems. A new feature is emphasis upon the analysis of the flow profiles to be expected under different conditions of channel shape and grade, rather than upon examples of situations in which different types of profiles will form. The latter type of treatment, which has predominated in the literature, is of little direct use to the practicing engineer.

The text does not include a complete and exhaustive treatment of the subject, about which much yet remains to be known. On the other hand, certain related topics, not strictly within the scope defined by the title, are discussed. There is a chapter on the moving hydraulic jump. Another, on slowly varied flow, embraces the type of routing problems that can be treated as steady flow if the changes are taken into account in writing the equation of continuity.

The first half of the book contains much material accumulated throughout the life work of the senior author; the last half is primarily the work of the junior author.

Professor E. W. Lane assisted in the preparation of Chapter IX and read the entire manuscript, giving much valuable advice. Mr. J. C. Stevens read the manuscript and made many worth-while suggestions. The late Professor Floyd A. Nagler and the late David L. Yarnell assisted in the development of some of the subject matter that appears here for the first time. In the course of classroom discussion many of the authors' students have made contributions which are incorporated in the text. To all who have helped directly, and to the many contributors to the literature whose efforts have been drawn upon, the authors wish to express their sincere appreciation. Finally, they wish to thank the staff of John Wiley and Sons for its fine cooperation in preparing the manuscript for the press.

SHERMAN M. WOODWARD
CHESLEY J. POSEY

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CHAPTER 1

INTRODUCTION

The hydraulics of steady flow in open channels is a small but important part of the rapidly developing science of hydraulics. Ordinarily treated but briefly in texts on hydraulics, it has become a necessary part of the hydraulic engineer's equipment. The construction of canals, flood channels, and other open channel works of both large and small magnitude has proceeded rapidly in recent years. A knowledge of the hydraulics of open channel flow is essential if economical and safe designs are to be obtained.

A clear conception of the meaning of the many terms used in the study of the flow of water in open channels is of utmost importance for progress in understanding the subject. A larger number of fundamental variables is necessary than in the study of flow in closed conduits or pipes. It is therefore possible to devise more derived variables. Many of these, such as velocity head, total head, and hydraulic radius, are essential. The use of too many derived variables, however, is confusing. Only those which have proven significant, and have been generally accepted, will be used in this book.

Any elongated depression through which water flows may be termed a channel. The flow is said to be open channel flow if the water has a free surface. Thus the flow in a pipe flowing part full is open channel flow. The sides and bottom of the channel are considered to be impervious. If the cross section of the channel does not change along its length, and the channel is straight in alignment and on constant grade, it is said to be a *uniform* channel. Natural watercourses are never truly uniform, but if exceptionally regular, they may be considered to be uniform for some purposes. The grade of a uniform channel is its slope, referred to the horizontal.

A transverse cross section taken at right angles to the axis of the channel is usually called, briefly, a *section*. If the channel is full just to the point of overflowing, the area of the cross section is the " area flowing full." Areas at lesser depths are referred to the particular depth or stage for definiteness. Thus " area at 10-foot depth " means the cross-sectional area of flow when the depth is 10 feet. It is obvious that the area, a single numerical measure, is insufficient to characterize com-

pletely the section. The only other measure that is generally used is the *hydraulic radius*, which is equal to the quotient of the area divided by the wetted perimeter. It has the dimension of a depth, and in very wide channels becomes nearly equal to the average depth. The hydraulic radius is not an entirely satisfactory measure, as is illustrated by the common procedure of dividing a flooded river valley into parts, and computing the hydraulic radii of the main channel and overbank portions separately.

The water surface in a channel is level transversely, if oscillations due to small waves are averaged out, except where there is a bend in the channel, or where there are local disturbances caused by a change in cross section. The water surface usually has a grade downward in the direction of flow.

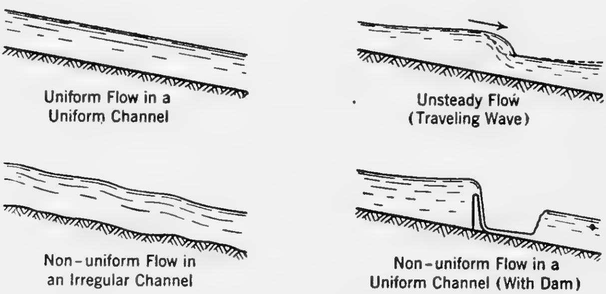


Fig. 101. Examples of the Different Types of Flow.

When the grade of the water surface in a uniform channel is the same as that of the bottom of the channel, there is said to be *uniform flow.* When the water surface is not parallel to the bottom, the flow is said to be *varied flow.* If the water surface elevation at every section remains the same, that is, does not change with respect to time, the flow is *steady.* The flow is *unsteady* when stages rise or fall, as during the passage of a flood wave. Varied flow may be steady or unsteady, but uniform flow is necessarily steady, in an open channel.

The vertical distance from the bottom of the channel to the water surface is the depth of flow. If the channel is uniform, the grade line of the bottom of the channel forms the most convenient base line from which to refer vertical measurements. If the channel is not uniform, and the profile of its bottom is not a straight line, it is usually preferable to refer all vertical measurements to a horizontal datum.

The volume of water passing a given section per unit of time is called the *discharge.* If the flow is steady the discharge is the same at all

sections along the channel, and remains constant with respect to time. The discharge divided by the area at a section gives the average velocity at the section, or more briefly, the velocity. Open channel flow is turbulent except in very small channels, so that the velocity of the different filaments in a section will usually show considerable variation, both in magnitude and in direction.

The velocity head at a section is equal to the square of the average velocity, divided by twice the acceleration of gravity. It is equal to the distance through which a freely falling body would have to fall from rest, under the influence of gravity alone, in order to acquire a velocity equal to the average velocity. The velocity heads of the individual filaments will vary considerably above and below the velocity head based on the average velocity, and it should be noted that the average of the velocity heads will be somewhat greater than the average-velocity head. This is important in precise investigations of energy loss, but is unimportant for practical hydraulic work, except in the few cases where the velocity distribution is markedly non-uniform.

The sum of the elevation head, measured above a fixed datum plane, and the pressure head, measured above atmospheric, is often called the *static head*. The static head at every point in a cross section of an open channel in which the flow is essentially parallel will be the same, even though the velocity head varies from filament to filament.

Another derived variable of great significance in open channel flow is that which represents the energy of the flow.[1] As a matter of convenience it is customary to approximate the energy of the stream by the *total head*, which is equal to the depth of flow plus the velocity head corresponding to the average velocity. The total head may be referred to the bottom of the channel, if it is a uniform channel, or to a convenient horizontal datum, if it is an irregular channel. A longitudinal profile of the elevation of the total head is called the *total head line*, or *energy line*.

[1] Use of the word "energy" in this connection is not logical and is likely to be misleading, but it has been well established by custom. The erroneous implication is that elevation head, pressure head, and velocity head each represent energy. Though a certain quantity of water may actually possess energy (the ability to do work) by virtue of its elevation or velocity, it cannot of itself possess energy because of being under pressure. The energy corresponding to its pressure head is only available when the situation is such that the surrounding fluid will close in, constantly maintaining full pressure, as the water is being removed to a region of lower pressure. Compare, for example, the work that could be obtained in releasing a pound of water from behind a dam with that obtainable in releasing a pound of water from a strong steel container which it completely fills, the initial and final pressures being the same in each case.

Steady uniform flow. The Manning and Kutter formulas, which are the open channel velocity or "friction" formulas most frequently used in the United States, apply only to steady, uniform flow. The channel should be uniform, and the slope of the water surface should be constant and the same as that of the bottom of the channel. The requirement of parallelism of water surface and channel bottom is the more important. The effect of channel irregularities may be taken into account, to a certain extent, in estimating the roughness, but lack of parallelism of the water surface and the general grade line of the bottom of the channel may cause direct application of the friction formulas to give grossly inaccurate results. Reasons for this, and methods for computing non-uniform flow, will be explained in later chapters.

The Manning formula is

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad [101]$$

The Ganguillet-Kutter formula, usually known as the Kutter formula, gives the value of the coefficient C in Chezy's formula

$$V = C\sqrt{RS} \qquad [102]$$

$$C = \frac{41.65 + \dfrac{0.00281}{S} + \dfrac{1.811}{n}}{1 + \dfrac{n}{\sqrt{R}}\left(41.65 + \dfrac{0.00281}{S}\right)} \qquad [103]$$

A discussion of these formulas, and the older formulas that they have supplanted, is given in Part IV of the *Technical Reports* of the Miami Conservancy District, also in H. W. King, *Handbook of Hydraulics*, and in many other sources. Recent studies of turbulence in open channel flow have suggested newer types of formulas, but for the present volume it will be assumed that the Manning and Kutter formulas are satisfactory.

Both formulas give the average velocity in terms of the hydraulic radius, the slope, and a roughness coefficient. In problems on uniform flow in a given channel, the quantities that need to be solved for may be (1) the velocity or the discharge, (2) the roughness coefficient, (3) the slope, or (4) the depth of flow. The shape of the cross section of the channel is assumed to be known.

Solution of the first three types of problems is straightforward, involving no especial difficulty except selection of the proper value of the roughness coefficient. Table 101 may be used as a guide. A detailed discussion of the values of the roughness coefficient to be used over a wide

range of circumstances is beyond the scope of this book. If more information is needed, the references listed in Table 101 should be consulted.

TABLE 101

Values of "n" for use in Manning's or Kutter's Formulas

Compiled from various sources. For more detailed information, see F. C. Scobey, "Flow of Water in Flumes," U.S.D.A. *Tech. Bull.* 393, and "The Flow of Water in Irrigation and Similar Canals," U.S.D.A. *Tech. Bull.* 652, also C. E. Ramser, "Flow of Water in Drainage Channels," U.S.D.A. *Tech. Bull.* 129, and Ivan E. Houk, "Calculation of Flow in Open Channels," Miami Conservancy District, *Technical Reports*, Part IV.

0.009 and 0.010	Very smooth and true surfaces, without projections. Clean new glass, pyralin, or brass, with straight alignment.
0.011 and 0.012	Smoothest clean wood, metal, or concrete surfaces, without projections, and with straight alignment.
0.013	Smooth wood, metal, or concrete surfaces without projections, free from algae or insect growth, and with reasonably straight alignment.
0.014	Good wood, metal, or concrete surfaces with very small projections, with some curvature, with slight insect or algae growth, or with slight gravel deposition. Shot concrete surfaced with troweled mortar.
0.015	Wood with algae and moss growth, concrete with smooth sides but roughly troweled or shot bottom, metal with shallow projections. Same with smoother surface but excessive curvature.
0.016	Metal flumes with large projections into the section. Wood or concrete with heavy algae, or moss growths.
0.017	Shot concrete, not troweled, but fairly uniform.
0.018–0.025	Metal flumes with large projections into the section and excessive curvature, growths, or accumulated debris.
0.016–0.017	Smoothest natural earth channels, free from growths, with straight alignment.
0.020	Smooth natural earth, free from growths, little curvature. Very large canals in good condition.
0.0225	Average, well-constructed, moderate-sized earth canal in good condition.
0.025	Very small earth canals or ditches in good condition, or larger canals with some growth on banks or scattered cobbles in bed.
0.030	Canals with considerable aquatic growth. Rock cuts, based on average actual section. Natural streams with good alignment, fairly constant section. Large floodway channels, well maintained.
0.035	Canals half choked with moss growth. Cleared but not continuously maintained floodways.
0.040–0.050	Mountain streams in clean loose cobbles. Rivers with variable section and some vegetation growing in banks. Canals with very heavy aquatic growths.
0.050–0.150	Natural streams of varying roughness and alignment. The highest values for extremely bad alignment, deep pools, and vegetation, or for floodways with heavy stand of timber and underbrush.

Determination of the depth of flow, with the discharge, roughness, and slope given, is indirect for most types of cross sections. The problem may be solved by assuming values for the depth and computing corresponding values of the discharge, repeating with different values for the depth until the computed value of the discharge agrees with the given value. Extra computation may be avoided if a synthetic rating-curve is drawn, plotting the assumed values of the depth against the corresponding values of the discharge. The use of logarithmic scales will simplify drawing the curve, for when plotted logarithmically the points tend to lie on a straight line. If the Manning formula is to be used, it is sometimes better to plot $AR^{2/3}$ versus the depth. The desired value of the depth is that for which $AR^{2/3}$ equals the value of $Qn/1.486S^{1/2}$ computed from the given discharge, roughness, and slope. This procedure is distinctly advantageous if the given values of S and n are likely to be revised, as often happens in design work. With either method, no more of the curve need be drawn than is necessary to arrive at the required depth.

FIG. 102. Overbank Flow.

The most complicated type of cross section, with regard to the computation of uniform flow, is that in which part of the flow is in a deep central channel, and part is in shallow side sections, as shown in Fig. 102. Computations based on the hydraulic radius of the entire cross section will be in error. Better results are obtained by computing the flow in the side and central portions separately, considering the areas and wetted perimeters of each to end at an imaginary vertical line above the bank of the submerged main channel.

Certain simple geometrical shapes are frequently used for canals and flumes. It is possible to simplify the determination of the depth of uniform flow in these sections. Consider first a triangular cross section with side slopes of z horizontal to 1.00 vertical. When the depth of flow is D the area is zD^2 and the wetted perimeter is $2D\sqrt{1 + z^2}$. Thus we may write

$$1.486AR^{2/3} = 1.486zD^2 \frac{z^{2/3}D^{4/3}}{(2D\sqrt{1 + z^2})^{2/3}}$$

$$= \left[0.936\sqrt[3]{\frac{z^5}{1 + z^2}}\right] D^{8/3}$$

Since by the Manning formula

$$Q = \frac{1.486}{n} AR^{2/3}S^{1/2}$$

it is seen that for a triangular channel

$$\frac{Qn}{S^{1/2}D^{8/3}} = \left[0.936\sqrt[3]{\frac{z^5}{1+z^2}}\right] \qquad [104]$$

The function in the brackets is easily evaluated.

z	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.5	3.0
$\frac{Qn}{S^{1/2}D^{8/3}}$	0.091	0.274	0.500	0.744	0.993	1.24	1.49	1.74	2.23	2.71

With this function known, the determination of the depth of flow becomes direct. Assume, for example, that $Q = 2$ c.f.s., $n = 0.014$, $S = 0.02$, and $z = 1.25$. Substitution in equation (104), using the tabulated value of the function in the brackets corresponding to $z = 1.25$, gives

$$D^{8/3} = \frac{2 \times 0.014}{0.142 \times 0.992} = 0.199$$

from which

$$D = 0.55 \text{ foot}$$

An equation similar to equation (104) may be obtained for other simple geometrical shapes, but the quantity in the brackets will usually be a function of more than one variable. Tables 102 to 104 give this function for trapezoidal, circular, and round-bottomed sections, permitting a quick solution for any of the variables listed. The table headings are self-explanatory.

Illustrative Example

Find the depth of flow for a discharge of 160 c.f.s. in a trapezoidal channel with 8 ft. bottom width, side slopes of 1 on 1, and a uniform bottom slope of 2 feet per thousand, if the coefficient of roughness may be taken as 0.017.

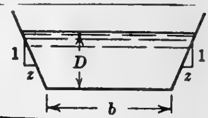
Summarizing the given information, $Q = 160$, $b = 8$, $z = 1$, $S = 0.002$, and $n = 0.017$. Since b is known, the use of Table 102A is indicated.

$$\frac{Qn}{b^{8/3}S^{1/2}} = \frac{160 \times 0.017}{256 \times 0.0447} = 0.238$$

Opposite this value in the column $z = 1$ of Table 102A, we find that $D/b = 0.326$. Hence D, the depth of flow, is $8 \times 0.326 = 2.61$ feet.

TABLE 102A

Uniform Flow in Trapezoidal Channels by Manning's Formula



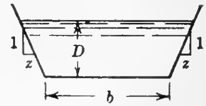
D/b*	Values of $\frac{Qn}{b^{8/3}S^{1/2}}$										
	$z = 0$	$z = \frac{1}{4}$	$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{4}$	$z = 1\frac{1}{2}$	$z = 2$	$z = 2\frac{1}{2}$	$z = 3$	$z = 4$
.02	.00213	.00215	.00216	.00217	.00218	.00219	.00220	.00221	.00222	.00223	.00225
.03	.00414	.00419	.00423	.00426	.00429	.00431	.00433	.00437	.00440	.00443	.00449
.04	.00661	.00670	.00679	.00685	.00690	.00696	.00700	.00707	.00715	.00722	.00735
.05	.00947	.00964	.00980	.00991	.0100	.0101	.0102	.0103	.0104	.0106	.0109
.06	.0127	.0130	.0132	.0134	.0136	.0137	.0138	.0141	.0143	.0145	.0149
.07	.0162	.0166	.0170	.0173	.0176	.0177	.0180	.0183	.0186	.0190	.0196
.08	.0200	.0206	.0211	.0215	.0219	.0222	.0225	.0231	.0235	.0240	.0250
.09	.0240	.0249	.0256	.0262	.0267	.0271	.0275	.0282	.0289	.0296	.0310
.10	.0283	.0294	.0305	.0311	.0318	.0324	.0329	.0339	.0348	.0358	.0375
.11	.0329	.0342	.0354	.0364	.0373	.0380	.0387	.0400	.0413	.0424	.0448
.12	.0376	.0393	.0408	.0420	.0431	.0441	.0450	.0466	.0482	.0497	.0527
.13	.0425	.0446	.0464	.0480	.0493	.0505	.0516	.0537	.0556	.0575	.0613
.14	.0476	.0501	.0524	.0542	.0559	.0573	.0587	.0612	.0636	.0659	.0705
.15	.0528	.0559	.0585	.0608	.0628	.0645	.0662	.0692	.0721	.0749	.0805
.16	.0582	.0619	.0650	.0676	.0699	.0720	.0740	.0776	.0811	.0845	.0912
.17	.0638	.0680	.0717	.0748	.0775	.0800	.0823	.0867	.0907	.0947	.103
.18	.0695	.0744	.0786	.0822	.0854	.0883	.0910	.0961	.101	.105	.115
.19	.0753	.0809	.0857	.0900	.0936	.0970	.100	.106	.112	.117	.128
.20	.0813	.0875	.0932	.0979	.102	.106	.110	.116	.123	.129	.141
.21	.0873	.0944	.101	.106	.111	.115	.120	.127	.134	.142	.156
.22	.0935	.101	.109	.115	.120	.125	.130	.139	.147	.155	.171
.23	.0997	.109	.117	.124	.130	.135	.141	.151	.160	.169	.187
.24	.106	.116	.125	.133	.139	.146	.152	.163	.173	.184	.204
.25	.113	.124	.133	.142	.150	.157	.163	.176	.187	.199	.222
.26	.119	.131	.142	.152	.160	.168	.175	.189	.202	.215	.241
.27	.126	.139	.151	.162	.171	.180	.188	.203	.218	.232	.260
.28	.133	.147	.160	.172	.182	.192	.201	.217	.234	.249	.281
.29	.139	.155	.170	.182	.193	.204	.214	.232	.250	.267	.302
.30	.146	.163	.179	.193	.205	.217	.227	.248	.267	.286	.324
.31	.153	.172	.189	.204	.217	.230	.242	.264	.285	.306	.347
.32	.160	.180	.199	.215	.230	.243	.256	.281	.304	.327	.371
.33	.167	.189	.209	.227	.243	.257	.271	.298	.323	.348	.396
.34	.174	.198	.219	.238	.256	.272	.287	.315	.343	.369	.422
.35	.181	.207	.230	.251	.270	.287	.303	.334	.363	.392	.450
.36	.190	.216	.241	.263	.283	.302	.319	.353	.384	.416	.477
.37	.196	.225	.251	.275	.297	.317	.336	.372	.406	.440	.507
.38	.203	.234	.263	.289	.311	.333	.354	.392	.429	.465	.536
.39	.210	.244	.274	.301	.326	.349	.371	.412	.452	.491	.568
.40	.218	.254	.286	.314	.341	.366	.389	.433	.476	.518	.600
.41	.225	.263	.297	.328	.357	.383	.408	.455	.501	.545	.634
.42	.233	.279	.310	.342	.373	.401	.427	.478	.526	.574	.668
.43	.241	.282	.321	.356	.389	.418	.447	.501	.553	.604	.703
.44	.249	.292	.334	.371	.405	.437	.467	.524	.579	.634	.739
.45	.256	.303	.346	.385	.422	.455	.487	.548	.607	.665	.778
.46	.263	.313	.359	.401	.439	.475	.509	.574	.635	.696	.816
.47	.271	.323	.371	.417	.457	.494	.530	.600	.665	.729	.856
.48	.279	.333	.384	.432	.475	.514	.552	.626	.695	.763	.897
.49	.287	.345	.398	.448	.492	.534	.575	.652	.725	.797	.939
.50	.295	.356	.411	.463	.512	.556	.599	.679	.758	.833	.983
.52	.310	.377	.438	.496	.548	.599	.646	.735	.820	.906	1.07
.54	.327	.398	.468	.530	.590	.644	.696	.795	.891	.984	1.17
.56	.343	.421	.496	.567	.631	.690	.748	.856	.963	1.07	1.27
.58	.359	.444	.526	.601	.671	.739	.802	.922	1.04	1.15	1.37
.60	.375	.468	.556	.640	.717	.789	.858	.988	1.12	1.24	1.49
.62	.391	.492	.590	.679	.763	.841	.917	1.06	1.20	1.33	1.60
.64	.408	.516	.620	.718	.809	.894	.976	1.13	1.28	1.43	1.72

* For D/b less than 0.04, use of the assumption $R = D$ is more convenient and more accurate than interpolation in the table.

TABLE 102A (*Continued*)

Uniform Flow in Trapezoidal Channels by Manning's Formula

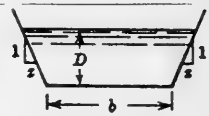
Values of $\frac{Qn}{b^{8/3}S^{1/2}}$



D/b	$z = 0$	$z = \frac{1}{4}$	$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{4}$	$z = 1\frac{1}{2}$	$z = 2$	$z = 2\frac{1}{2}$	$z = 3$	$z = 4$
.66	.424	.541	.653	.759	.858	.951	1.04	1.21	1.37	1.53	1.85
.68	.441	.566	.687	.801	.908	1.01	1.10	1.29	1.47	1.64	1.98
.70	.457	.591	.722	.842	.958	1.07	1.17	1.37	1.56	1.75	2.12
.72	.474	.617	.757	.887	1.01	1.13	1.24	1.45	1.66	1.87	2.27
.74	.491	.644	.793	.932	1.07	1.19	1.31	1.55	1.77	1.98	2.41
.76	.508	.670	.830	.981	1.12	1.26	1.39	1.64	1.88	2.11	2.57
.78	.525	.698	.868	1.03	1.18	1.32	1.46	1.73	1.98	2.24	2.73
.80	.542	.725	.906	1.08	1.24	1.40	1.54	1.83	2.10	2.37	2.90
.82	.559	.753	.945	1.13	1.30	1.47	1.63	1.93	2.22	2.51	3.07
.84	.576	.782	.985	1.18	1.36	1.54	1.71	2.03	2.34	2.65	3.25
.86	.593	.810	1.03	1.23	1.43	1.61	1.79	2.14	2.47	2.80	3.44
.88	.610	.839	1.07	1.29	1.49	1.69	1.88	2.25	2.60	2.95	3.63
.90	.627	.871	1.11	1.34	1.56	1.77	1.98	2.36	2.74	3.11	3.83
.92	.645	.898	1.15	1.40	1.63	1.86	2.07	2.48	2.88	3.27	4.04
.94	.662	.928	1.20	1.46	1.70	1.94	2.16	2.60	3.03	3.43	4.25
.96	.680	.960	1.25	1.52	1.78	2.03	2.27	2.73	3.17	3.61	4.48
.98	.697	.991	1.29	1.58	1.85	2.11	2.37	2.85	3.33	3.79	4.70
1.00	.714	1.02	1.33	1.64	1.93	2.21	2.47	2.99	3.48	3.97	4.93
1.05	.759	1.10	1.46	1.80	2.13	2.44	2.75	3.33	3.90	4.45	5.55
1.10	.802	1.19	1.58	1.97	2.34	2.69	3.04	3.70	4.34	4.96	6.21
1.15	.846	1.27	1.71	2.14	2.56	2.96	3.34	4.09	4.82	5.52	6.91
1.20	.891	1.36	1.85	2.33	2.79	3.24	3.68	4.50	5.32	6.11	7.68
1.25	.936	1.45	1.99	2.52	3.04	3.54	4.03	4.95	5.86	6.73	8.48
1.30	.980	1.54	2.14	2.73	3.30	3.85	4.39	5.42	6.42	7.39	9.34
1.35	1.02	1.64	2.29	2.94	3.57	4.18	4.76	5.90	7.01	8.10	10.2
1.40	1.07	1.74	2.45	3.16	3.85	4.52	5.18	6.43	7.65	8.83	11.2
1.45	1.11	1.84	2.61	3.39	4.15	4.88	5.60	6.98	8.30	9.62	12.2
1.50	1.16	1.94	2.78	3.63	4.46	5.26	6.04	7.55	9.02	10.4	13.3
1.55	1.20	2.05	2.96	3.88	4.78	5.65	6.50	8.14	9.74	11.3	14.4
1.60	1.25	2.15	3.14	4.14	5.12	6.06	6.99	8.79	10.5	12.2	15.6
1.65	1.30	2.27	3.33	4.41	5.47	6.49	7.50	9.42	11.3	13.2	16.8
1.70	1.34	2.38	3.52	4.69	5.83	6.94	8.02	10.1	12.2	14.2	18.1
1.75	1.39	2.50	3.73	4.98	6.21	7.41	8.57	10.9	13.0	15.2	19.5
1.80	1.43	2.62	3.93	5.28	6.60	7.89	9.13	11.6	14.0	16.3	20.9
1.85	1.48	2.74	4.15	5.59	7.01	8.40	9.75	12.4	15.0	17.4	22.4
1.90	1.52	2.86	4.36	5.91	7.43	8.91	10.4	13.2	15.9	18.7	24.0
1.95	1.57	2.99	4.59	6.24	7.87	9.46	11.0	14.0	17.0	19.9	25.6
2.00	1.61	3.12	4.83	6.58	8.32	10.0	11.7	14.9	18.0	21.1	27.2
2.10	1.71	3.39	5.31	7.30	9.27	11.2	13.1	16.8	20.3	23.9	30.8
2.20	1.79	3.67	5.82	8.06	10.3	12.5	14.6	18.7	22.8	26.8	34.7
2.30	1.89	3.96	6.36	8.86	11.3	13.8	16.2	20.9	25.4	30.0	38.8
2.40	1.98	4.26	6.93	9.72	12.5	15.3	17.9	23.1	28.3	33.4	43.3
2.50	2.07	4.58	7.52	10.6	13.7	16.8	19.8	25.6	31.3	37.0	48.0
2.60	2.16	4.90	8.14	11.6	15.0	18.4	21.7	28.2	34.5	40.8	53.0
2.70	2.26	5.24	8.80	12.6	16.3	20.1	23.8	31.0	37.9	44.8	58.4
2.80	2.35	5.59	9.49	13.6	17.8	21.9	25.9	33.8	41.6	49.1	64.0
2.90	2.44	5.95	10.2	14.7	19.3	23.8	28.2	36.9	45.3	53.7	70.1
3.00	2.53	6.33	11.0	15.9	20.9	25.8	30.6	40.1	49.4	58.4	76.4
3.20	2.72	7.12	12.5	18.3	24.2	30.1	35.8	47.1	58.0	68.9	90.3
3.40	2.90	7.97	14.2	21.0	27.9	34.8	41.5	54.6	67.7	80.2	105
3.60	3.09	8.86	16.1	24.0	32.0	39.9	47.8	63.0	78.2	92.8	122
3.80	3.28	9.81	18.1	27.1	36.3	45.5	54.6	72.4	89.6	107	141
4.00	3.46	10.8	20.2	30.5	41.1	51.6	61.9	82.2	102	122	160
4.50	3.92	13.5	26.2	40.1	54.5	68.8	82.9	111	136	164	217
5.00	4.39	16.7	33.1	51.5	70.3	89.2	108	145	181	216	287

TABLE 102B

Uniform Flow in Trapezoidal Channels by Manning's Formula



Values of $\frac{Qn}{D^{8/3}S^{1/2}}$

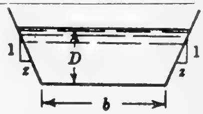
D/b*	$z = 0$	$z = \frac{1}{4}$	$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{4}$	$z = 1\frac{1}{2}$	$z = 2$	$z = 2\frac{1}{2}$	$z = 3$	$z = 4$
.01	146.7	147.2	147.6	148.0	148.3	148.6	148.8	149.2	149.5	149.9	150.5
.02	72.4	72.9	73.4	73.7	74.0	74.3	74.5	74.9	75.3	75.6	76.3
.03	47.6	48.2	48.6	49.0	49.3	49.5	49.8	50.2	50.6	50.9	51.6
.04	35.3	35.8	36.3	36.6	36.9	37.2	37.4	37.8	38.2	38.6	39.3
.05	27.9	28.4	28.9	29.2	29.5	29.8	30.0	30.5	30.8	31.2	32.0
.06	23.0	23.5	23.9	24.3	24.6	24.8	25.1	25.5	25.9	26.3	27.1
.07	19.45	19.97	20.4	20.8	21.1	21.3	21.6	22.0	22.4	22.8	23.6
.08	16.82	17.34	17.73	18.13	18.43	18.70	18.95	19.40	19.82	20.2	21.0
.09	14.78	15.29	15.72	16.08	16.39	16.66	16.91	17.36	17.79	18.21	19.04
.10	13.16	13.66	14.14	14.44	14.75	15.02	15.28	15.74	16.17	16.60	17.43
.11	11.83	12.33	12.76	13.11	13.42	13.69	13.94	14.41	14.85	15.28	16.13
.12	10.73	11.23	11.65	12.00	12.31	12.59	12.84	13.31	13.75	14.19	15.05
.13	9.80	10.29	10.71	11.06	11.37	11.65	11.90	12.38	12.83	13.26	14.13
.14	9.00	9.49	9.91	10.26	10.57	10.85	11.10	11.58	12.03	12.48	13.35
.15	8.32	8.80	9.21	9.57	9.88	10.16	10.42	10.89	11.35	11.80	12.68
.16	7.72	8.20	8.61	8.96	9.27	9.55	9.81	10.29	10.75	11.20	12.09
.17	7.19	7.67	8.08	8.43	8.74	9.02	9.28	9.77	10.23	10.68	11.57
.18	6.73	7.20	7.61	7.96	8.27	8.55	8.81	9.30	9.76	10.21	11.11
.19	6.31	6.78	7.18	7.54	7.85	8.13	8.39	8.88	9.35	9.80	10.70
.20	5.94	6.40	6.81	7.16	7.47	7.75	8.01	8.50	8.97	9.43	10.33
.21	5.60	6.06	6.47	6.82	7.13	7.41	7.67	8.16	8.63	9.09	10.00
.22	5.30	5.75	6.16	6.50	6.82	7.10	7.36	7.86	8.33	8.79	9.70
.23	5.02	5.47	5.87	6.22	6.53	6.81	7.08	7.58	8.05	8.51	9.43
.24	4.77	5.22	5.62	5.96	6.27	6.56	6.82	7.32	7.79	8.26	9.18
.25	4.54	4.98	5.38	5.73	6.04	6.32	6.58	7.08	7.56	8.03	8.95
.26	4.32	4.77	5.16	5.51	5.82	6.10	6.37	6.87	7.35	7.81	8.74
.27	4.13	4.57	4.96	5.31	5.62	5.90	6.16	6.67	7.15	7.62	8.54
.28	3.95	4.38	4.77	5.12	5.43	5.71	5.98	6.48	6.96	7.43	8.36
.29	3.78	4.21	4.60	4.95	5.25	5.54	5.81	6.31	6.79	7.26	8.19
.30	3.62	4.05	4.44	4.78	5.09	5.38	5.64	6.15	6.63	7.10	8.04
.31	3.48	3.90	4.29	4.63	4.94	5.23	5.49	6.00	6.48	6.96	7.89
.32	3.34	3.76	4.15	4.49	4.80	5.08	5.35	5.86	6.34	6.82	7.75
.33	3.21	3.64	4.02	4.36	4.67	4.95	5.22	5.73	6.21	6.69	7.62
.34	3.09	3.51	3.89	4.23	4.54	4.83	5.09	5.60	6.09	6.56	7.50
.35	2.98	3.40	3.78	4.12	4.43	4.71	4.98	5.49	5.97	6.45	7.39
.36	2.88	3.29	3.67	4.01	4.32	4.60	4.87	5.38	5.86	6.34	7.28
.37	2.78	3.19	3.56	3.90	4.21	4.49	4.76	5.27	5.76	6.24	7.18
.38	2.68	3.09	3.47	3.81	4.11	4.40	4.67	5.17	5.66	6.14	7.08
.39	2.59	3.00	3.37	3.71	4.02	4.30	4.57	5.08	5.57	6.05	6.99
.40	2.51	2.92	3.29	3.62	3.93	4.21	4.48	4.99	5.48	5.96	6.91
.41	2.43	2.84	3.20	3.54	3.85	4.13	4.40	4.91	5.40	5.88	6.83
.42	2.36	2.76	3.13	3.46	3.77	4.05	4.32	4.83	5.32	5.80	6.75
.43	2.29	2.68	3.05	3.38	3.69	3.97	4.24	4.76	5.25	5.73	6.67
.44	2.22	2.61	2.98	3.31	3.62	3.90	4.17	4.68	5.17	5.66	6.60
.45	2.15	2.55	2.91	3.24	3.55	3.83	4.10	4.61	5.11	5.59	6.54
.46	2.09	2.48	2.85	3.18	3.48	3.77	4.04	4.55	5.04	5.52	6.47
.47	2.03	2.42	2.78	3.12	3.42	3.70	3.97	4.49	4.98	5.46	6.41
.48	1.977	2.36	2.72	3.06	3.36	3.64	3.91	4.43	4.92	5.40	6.35
.49	1.923	2.31	2.67	3.00	3.30	3.58	3.85	4.37	4.86	5.34	6.29
.50	1.872	2.26	2.61	2.94	3.25	3.53	3.80	4.31	4.81	5.29	6.24
.52	1.777	2.16	2.51	2.84	3.14	3.43	3.70	4.21	4.70	5.19	6.14
.54	1.689	2.06	2.42	2.74	3.05	3.33	3.60	4.11	4.61	5.09	6.04
.56	1.608	1.977	2.33	2.66	2.96	3.24	3.51	4.02	4.52	5.00	5.96
.58	1.533	1.900	2.25	2.57	2.87	3.16	3.43	3.94	4.44	4.92	5.87
.60	1.464	1.827	2.17	2.50	2.80	3.08	3.35	3.86	4.36	4.85	5.80
.62	1.400	1.760	2.11	2.43	2.73	3.01	3.28	3.79	4.29	4.77	5.73
.64	1.340	1.697	2.04	2.36	2.66	2.94	3.21	3.72	4.22	4.71	5.66

* For D/b less than 0.04, use of the assumption $R = D$ is more convenient and more accurate than interpolation in the table.

TABLE 102B (*Continued*)

Uniform Flow in Trapezoidal Channels by Manning's Formula

Values of $\frac{Qn}{D^{8/3}S^{1/2}}$



D/b	$z = 0$	$z = \frac{1}{4}$	$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{4}$	$z = 1\frac{1}{2}$	$z = 2$	$z = 2\frac{1}{2}$	$z = 3$	$z = 4$
.66	1.285	1.638	1.979	2.30	2.60	2.88	3.15	3.66	4.16	4.64	5.60
.68	1.234	1.583	1.922	2.24	2.54	2.82	3.09	3.60	4.10	4.59	5.54
.70	1.184	1.531	1.868	2.18	2.48	2.76	3.03	3.55	4.04	4.53	5.49
.72	1.139	1.482	1.818	2.13	2.43	2.71	2.98	3.49	3.99	4.48	5.44
.74	1.096	1.437	1.770	2.08	2.38	2.66	2.93	3.45	3.94	4.43	5.39
.76	1.056	1.393	1.725	2.04	2.33	2.61	2.88	3.40	3.90	4.38	5.34
.78	1.018	1.353	1.683	1.998	2.29	2.57	2.84	3.35	3.85	4.34	5.30
.80	.982	1.315	1.642	1.954	2.25	2.53	2.80	3.31	3.81	4.30	5.26
.82	.949	1.278	1.604	1.916	2.21	2.49	2.76	3.27	3.77	4.26	5.22
.84	.917	1.245	1.568	1.886	2.17	2.45	2.72	3.23	3.73	4.22	5.18
.86	.887	1.211	1.534	1.843	2.14	2.41	2.68	3.20	3.69	4.18	5.14
.88	.858	1.180	1.501	1.810	2.10	2.38	2.65	3.16	3.66	4.15	5.11
.90	.831	1.153	1.470	1.777	2.07	2.35	2.62	3.13	3.63	4.12	5.07
.92	.805	1.122	1.441	1.747	2.04	2.32	2.58	3.10	3.60	4.08	5.04
.94	.781	1.095	1.413	1.718	2.01	2.29	2.55	3.07	3.57	4.05	5.01
.96	.758	1.070	1.396	1.690	1.981	2.26	2.53	3.04	3.54	4.03	4.99
.98	.736	1.046	1.360	1.663	1.954	2.23	2.50	3.01	3.51	4.00	4.96
1.00	.714	1.022	1.335	1.638	1.928	2.21	2.47	2.99	3.48	3.97	4.93
1.05	.666	.969	1.278	1.579	1.871	2.14	2.41	2.92	3.42	3.91	4.87
1.10	.622	.920	1.226	1.525	1.813	2.09	2.36	2.87	3.37	3.85	4.82
1.15	.583	.876	1.178	1.477	1.763	2.04	2.30	2.82	3.32	3.80	4.76
1.20	.548	.836	1.136	1.432	1.717	1.993	2.26	2.77	3.27	3.76	4.72
1.25	.516	.800	1.098	1.392	1.676	1.950	2.22	2.73	3.23	3.71	4.68
1.30	.487	.767	1.062	1.354	1.638	1.912	2.18	2.69	3.19	3.67	4.64
1.35	.460	.736	1.028	1.320	1.603	1.876	2.14	2.65	3.15	3.64	4.60
1.40	.436	.708	.998	1.288	1.570	1.843	2.11	2.62	3.12	3.60	4.57
1.45	.414	.682	.970	1.259	1.540	1.812	2.08	2.59	3.08	3.57	4.53
1.50	.393	.658	.944	1.231	1.512	1.784	2.05	2.56	3.06	3.54	4.51
1.55	.374	.636	.920	1.206	1.486	1.757	2.02	2.53	3.03	3.52	4.48
1.60	.357	.615	.897	1.182	1.461	1.731	1.995	2.51	3.00	3.49	4.45
1.65	.341	.596	.876	1.160	1.438	1.708	1.972	2.48	2.98	3.47	4.43
1.70	.325	.578	.856	1.139	1.416	1.686	1.949	2.46	2.96	3.44	4.40
1.75	.312	.561	.838	1.119	1.396	1.666	1.928	2.44	2.93	3.42	4.38
1.80	.298	.546	.820	1.101	1.377	1.646	1.905	2.42	2.91	3.40	4.36
1.85	.286	.531	.804	1.083	1.359	1.628	1.890	2.40	2.90	3.38	4.34
1.90	.275	.517	.788	1.067	1.342	1.610	1.872	2.38	2.88	3.37	4.33
1.95	.264	.504	.773	1.051	1.326	1.594	1.856	2.36	2.86	3.35	4.31
2.00	.254	.491	.760	1.036	1.310	1.578	1.840	2.35	2.84	3.33	4.29
2.10	.236	.469	.734	1.009	1.282	1.549	1.811	2.32	2.81	3.30	4.26
2.20	.219	.448	.711	.984	1.256	1.523	1.784	2.29	2.79	3.27	4.24
2.30	.205	.430	.690	.962	1.233	1.499	1.760	2.27	2.76	3.25	4.21
2.40	.1919	.413	.671	.941	1.212	1.477	1.737	2.24	2.74	3.23	4.19
2.50	.1800	.398	.653	.922	1.192	1.457	1.717	2.22	2.72	3.21	4.17
2.60	.1693	.383	.637	.905	1.174	1.438	1.698	2.21	2.70	3.19	4.15
2.70	.1597	.371	.623	.889	1.157	1.422	1.681	2.19	2.68	3.17	4.13
2.80	.1508	.359	.609	.874	1.142	1.406	1.665	2.17	2.67	3.15	4.11
2.90	.1427	.348	.596	.861	1.128	1.391	1.650	2.16	2.65	3.14	4.10
3.00	.1354	.338	.585	.848	1.114	1.377	1.636	2.14	2.64	3.12	4.08
3.20	.1223	.320	.563	.825	1.090	1.353	1.611	2.12	2.61	3.10	4.06
3.40	.1111	.305	.545	.805	1.069	1.331	1.589	2.09	2.59	3.07	4.03
3.60	.1015	.291	.529	.787	1.050	1.312	1.569	2.07	2.57	3.05	4.01
3.80	.0932	.279	.514	.771	1.033	1.294	1.552	2.06	2.55	3.04	4.00
4.00	.0859	.268	.501	.757	1.019	1.279	1.536	2.04	2.53	3.02	3.98
4.50	.0711	.245	.474	.727	.987	1.246	1.502	2.01	2.50	2.98	3.94
5.00	.0601	.228	.453	.704	.962	1.220	1.476	1.979	2.47	2.96	3.92

TABLE 103

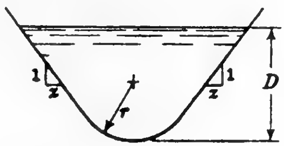
Uniform Flow in Circular Sections Flowing Partly Full

d = depth of flow
D = diameter of pipe
A = area of flow
R = hydraulic radius
Q = discharge in c.f.s. by Manning's formula
n = Manning's coefficient
S = slope of the channel bottom and of the water surface

$\frac{d}{D}$	$\frac{A}{D^2}$	$\frac{R}{D}$	$\frac{Qn}{D^{8/3}S^{1/2}}$	$\frac{Qn}{d^{8/3}S^{1/2}}$	$\frac{d}{D}$	$\frac{A}{D^2}$	$\frac{R}{D}$	$\frac{Qn}{D^{8/3}S^{1/2}}$	$\frac{Qn}{d^{8/3}S^{1/2}}$
0.01	0.0013	0.0066	0.00007	15.04	0.51	0.4027	0.2531	0.239	1.442
0.02	0.0037	0.0132	0.00031	10.57	0.52	0.4127	0.2562	0.247	1.415
0.03	0.0069	0.0197	0.00074	8.56	0.53	0.4227	0.2592	0.255	1.388
0.04	0.0105	0.0262	0.00138	7.38	0.54	0.4327	0.2621	0.263	1.362
0.05	0.0147	0.0325	0.00222	6.55	0.55	0.4426	0.2649	0.271	1.336
0.06	0.0192	0.0389	0.00328	5.95	0.56	0.4526	0.2676	0.279	1.311
0.07	0.0242	0.0451	0.00455	5.47	0.57	0.4625	0.2703	0.287	1.286
0.08	0.0294	0.0513	0.00604	5.09	0.58	0.4724	0.2728	0.295	1.262
0.09	0.0350	0.0575	0.00775	4.76	0.59	0.4822	0.2753	0.303	1.238
0.10	0.0409	0.0635	0.00967	4.49	0.60	0.4920	0.2776	0.311	1.215
0.11	0.0470	0.0695	0.01181	4.25	0.61	0.5018	0.2799	0.319	1.192
0.12	0.0534	0.0755	0.01417	4.04	0.62	0.5115	0.2821	0.327	1.170
0.13	0.0600	0.0813	0.01674	3.86	0.63	0.5212	0.2842	0.335	1.148
0.14	0.0668	0.0871	0.01952	3.69	0.64	0.5308	0.2862	0.343	1.126
0.15	0.0739	0.0929	0.0225	3.54	0.65	0.5404	0.2882	0.350	1.105
0.16	0.0811	0.0985	0.0257	3.41	0.66	0.5499	0.2900	0.358	1.084
0.17	0.0885	0.1042	0.0291	3.28	0.67	0.5594	0.2917	0.366	1.064
0.18	0.0961	0.1097	0.0327	3.17	0.68	0.5687	0.2933	0.373	1.044
0.19	0.1039	0.1152	0.0365	3.06	0.69	0.5780	0.2948	0.380	1.024
0.20	0.1118	0.1206	0.0406	2.96	0.70	0.5872	0.2962	0.388	1.004
0.21	0.1199	0.1259	0.0448	2.87	0.71	0.5964	0.2975	0.395	0.985
0.22	0.1281	0.1312	0.0492	2.79	0.72	0.6054	0.2987	0.402	0.965
0.23	0.1365	0.1364	0.0537	2.71	0.73	0.6143	0.2998	0.409	0.947
0.24	0.1449	0.1416	0.0585	2.63	0.74	0.6231	0.3008	0.416	0.928
0.25	0.1535	0.1466	0.0634	2.56	0.75	0.6319	0.3017	0.422	0.910
0.26	0.1623	0.1516	0.0686	2.49	0.76	0.6405	0.3024	0.429	0.891
0.27	0.1711	0.1566	0.0739	2.42	0.77	0.6489	0.3031	0.435	0.873
0.28	0.1800	0.1614	0.0793	2.36	0.78	0.6573	0.3036	0.441	0.856
0.29	0.1890	0.1662	0.0849	2.30	0.79	0.6655	0.3039	0.447	0.838
0.30	0.1982	0.1709	0.0907	2.25	0.80	0.6736	0.3042	0.453	0.821
0.31	0.2074	0.1756	0.0966	2.20	0.81	0.6815	0.3043	0.458	0.804
0.32	0.2167	0.1802	0.1027	2.14	0.82	0.6893	0.3043	0.463	0.787
0.33	0.2260	0.1847	0.1089	2.09	0.83	0.6969	0.3041	0.468	0.770
0.34	0.2355	0.1891	0.1153	2.05	0.84	0.7043	0.3038	0.473	0.753
0.35	0.2450	0.1935	0.1218	2.00	0.85	0.7115	0.3033	0.477	0.736
0.36	0.2546	0.1978	0.1284	1.958	0.86	0.7186	0.3026	0.481	0.720
0.37	0.2642	0.2020	0.1351	1.915	0.87	0.7254	0.3018	0.485	0.703
0.38	0.2739	0.2062	0.1420	1.875	0.88	0.7320	0.3007	0.488	0.687
0.39	0.2836	0.2102	0.1490	1.835	0.89	0.7384	0.2995	0.491	0.670
0.40	0.2934	0.2142	0.1561	1.797	0.90	0.7445	0.2980	0.494	0.654
0.41	0.3032	0.2182	0.1633	1.760	0.91	0.7504	0.2963	0.496	0.637
0.42	0.3130	0.2220	0.1705	1.724	0.92	0.7560	0.2944	0.497	0.621
0.43	0.3229	0.2258	0.1779	1.689	0.93	0.7612	0.2921	0.498	0.604
0.44	0.3328	0.2295	0.1854	1.655	0.94	0.7662	0.2895	0.498	0.588
0.45	0.3428	0.2331	0.1929	1.622	0.95	0.7707	0.2865	0.498	0.571
0.46	0.3527	0.2366	0.201	1.590	0.96	0.7749	0.2829	0.496	0.553
0.47	0.3627	0.2401	0.208	1.559	0.97	0.7785	0.2787	0.494	0.535
0.48	0.3727	0.2435	0.216	1.530	0.98	0.7817	0.2735	0.489	0.517
0.49	0.3827	0.2468	0.224	1.500	0.99	0.7841	0.2666	0.483	0.496
0.50	0.3927	0.2500	0.232	1.471	1.00	0.7854	0.2500	0.463	0.463

TABLE 104A

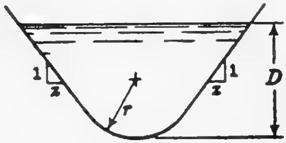
Uniform Flow in a Special Round-Bottomed Channel by Manning's Formula



D/r	Values of $\frac{Qn}{r^{8/3}S^{1/2}}$					D/r	Values of $\frac{Qn}{r^{8/3}S^{1/2}}$				
	$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{2}$	$z = 2$		$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{2}$	$z = 2$
0.02	0.00044	0.00044	0.00044	0.00044	0.00044	0.82	1.040	1.061	1.103	1.243	1.432
0.04	0.00198	0.00198	0.00198	0.00198	0.00198	0.84	1.088	1.112	1.160	1.313	1.517
0.06	0.00472	0.00472	0.00472	0.00472	0.00472	0.86	1.137	1.165	1.218	1.385	1.604
0.08	0.00877	0.00877	0.00877	0.00877	0.00877	0.88	1.187	1.219	1.278	1.459	1.695
0.10	0.01411	0.01411	0.01411	0.01411	0.01411	0.90	1.260	1.274	1.339	1.535	1.789
0.12	0.0208	0.0208	0.0208	0.0208	0.0208	0.92	1.289	1.331	1.402	1.614	1.886
0.14	0.0289	0.0289	0.0289	0.0289	0.0289	0.94	1.341	1.389	1.467	1.696	1.986
0.16	0.0384	0.0384	0.0384	0.0384	0.0384	0.96	1.395	1.448	1.534	1.780	2.09
0.18	0.0492	0.0492	0.0492	0.0492	0.0492	0.98	1.449	1.508	1.602	1.866	2.20
0.20	0.0614	0.0614	0.0614	0.0614	0.0614	1.00	1.504	1.570	1.671	1.954	2.31
0.22	0.0750	0.0750	0.0750	0.0750	0.0750	1.05	1.647	1.730	1.853	2.19	2.60
0.24	0.0899	0.0899	0.0899	0.0899	0.0899	1.10	1.794	1.899	2.05	2.44	2.91
0.26	0.106	0.106	0.106	0.106	0.107	1.15	1.950	2.08	2.25	2.70	3.24
0.28	0.124	0.124	0.124	0.124	0.125	1.20	2.11	2.26	2.47	2.99	3.60
0.30	0.143	0.143	0.143	0.143	0.145	1.25	2.28	2.46	2.69	3.29	3.98
0.32	0.163	0.163	0.163	0.163	0.167	1.30	2.45	2.66	2.93	3.60	4.38
0.34	0.185	0.185	0.185	0.185	0.191	1.35	2.63	2.87	3.18	3.94	4.81
0.36	0.208	0.208	0.208	0.208	0.216	1.40	2.81	3.09	3.45	4.29	5.26
0.38	0.233	0.233	0.233	0.234	0.244	1.45	3.01	3.33	3.72	4.67	5.74
0.40	0.257	0.257	0.257	0.261	0.274	1.50	3.20	3.56	4.00	5.05	6.23
0.42	0.284	0.284	0.284	0.288	0.305	1.55	3.41	3.82	4.31	5.47	6.77
0.44	0.312	0.312	0.312	0.319	0.338	1.60	3.62	4.08	4.63	5.91	7.33
0.46	0.342	0.342	0.342	0.349	0.374	1.65	3.84	4.35	4.95	6.36	7.91
0.48	0.371	0.371	0.371	0.383	0.412	1.70	4.06	4.63	5.30	6.84	8.53
0.50	0.403	0.403	0.403	0.417	0.452	1.75	4.29	4.92	5.65	7.33	9.17
0.52	0.435	0.435	0.435	0.455	0.495	1.80	4.53	5.22	6.02	7.85	9.84
0.54	0.468	0.468	0.470	0.493	0.539	1.85	4.77	5.52	6.41	8.39	10.55
0.56	0.504	0.504	0.506	0.534	0.586	1.90	5.02	5.85	6.80	8.95	11.28
0.58	0.539	0.539	0.543	0.577	0.635	1.95	5.28	6.18	7.22	9.54	12.04
0.60	0.576	0.576	0.582	0.621	0.687	2.0	5.55	6.53	7.64	10.14	12.83
0.62	0.613	0.614	0.622	0.667	0.742	2.1	6.10	7.25	8.55	11.43	14.52
0.64	0.652	0.653	0.663	0.715	0.798	2.2	6.68	8.01	9.51	12.81	16.34
0.66	0.691	0.694	0.706	0.766	0.858	2.3	7.29	8.82	10.53	14.30	18.29
0.68	0.732	0.735	0.751	0.818	0.920	2.4	7.93	9.68	11.63	15.88	20.4
0.70	0.773	0.778	0.797	0.872	0.985	2.5	8.60	10.58	12.78	17.57	22.6
0.72	0.815	0.822	0.844	0.929	1.052	2.6	9.29	11.54	14.01	19.37	25.0
0.74	0.858	0.868	0.893	0.987	1.122	2.7	10.03	12.54	15.31	21.27	27.5
0.76	0.902	0.914	0.943	1.048	1.195	2.8	10.79	13.59	16.67	23.29	30.2
0.78	0.947	0.962	0.995	1.111	1.271	2.9	11.58	14.69	18.11	25.43	33.1
0.80	0.993	1.010	1.048	1.176	1.350	3.0	12.41	15.86	19.62	27.68	36.1

TABLE 104B

Uniform Flow in a Special Round-Bottomed Channel by Manning's Formula



D/r	Values of $\frac{Qn}{D^{8/3}S^{1/2}}$					D/r	Values of $\frac{Qn}{D^{8/3}S^{1/2}}$				
	$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{2}$	$z = 2$		$z = \frac{1}{2}$	$z = \frac{3}{4}$	$z = 1$	$z = 1\frac{1}{2}$	$z = 2$
0.02	15.04	15.04	15.04	15.04	15.04	0.82	1.766	1.801	1.873	2.11	2.43
0.04	10.57	10.57	10.57	10.57	10.57	0.84	1.732	1.771	1.847	2.09	2.41
0.06	8.56	8.56	8.56	8.56	8.56	0.86	1.700	1.742	1.821	2.07	2.40
0.08	7.38	7.38	7.38	7.38	7.38	0.88	1.668	1.714	1.797	2.05	2.38
0.10	6.55	6.55	6.55	6.55	6.55	0.90	1.638	1.688	1.774	2.03	2.37
0.12	5.95	5.95	5.95	5.95	5.95	0.92	1.610	1.662	1.752	2.02	2.36
0.14	5.47	5.47	5.47	5.47	5.47	0.94	1.582	1.638	1.730	2.00	2.34
0.16	5.09	5.09	5.09	5.09	5.09	0.96	1.555	1.614	1.710	1.984	2.33
0.18	4.76	4.76	4.76	4.76	4.76	0.98	1.529	1.592	1.690	1.969	2.32
0.20	4.49	4.49	4.49	4.49	4.49	1.00	1.504	1.570	1.671	1.955	2.31
0.22	4.25	4.25	4.25	4.25	4.25	1.05	1.446	1.519	1.627	1.921	2.28
0.24	4.04	4.04	4.04	4.04	4.04	1.10	1.392	1.473	1.587	1.890	2.25
0.26	3.86	3.86	3.86	3.86	3.87	1.15	1.343	1.430	1.550	1.862	2.23
0.28	3.69	3.69	3.69	3.69	3.74	1.20	1.298	1.391	1.516	1.836	2.21
0.30	3.54	3.54	3.54	3.54	3.60	1.25	1.256	1.355	1.485	1.812	2.19
0.32	3.41	3.41	3.41	3.41	3.49	1.30	1.217	1.321	1.456	1.790	2.17
0.34	3.28	3.28	3.28	3.28	3.39	1.35	1.181	1.290	1.430	1.770	2.16
0.36	3.17	3.17	3.17	3.17	3.30	1.40	1.147	1.262	1.405	1.751	2.14
0.38	3.08	3.08	3.08	3.09	3.22	1.45	1.116	1.235	1.382	1.733	2.13
0.40	2.96	2.96	2.96	3.00	3.15	1.50	1.087	1.210	1.360	1.717	2.12
0.42	2.87	2.87	2.87	2.91	3.08	1.55	1.059	1.186	1.340	1.701	2.10
0.44	2.79	2.79	2.79	2.85	3.02	1.60	1.033	1.164	1.321	1.686	2.09
0.46	2.71	2.71	2.71	2.77	2.97	1.65	1.009	1.144	1.303	1.673	2.08
0.48	2.63	2.63	2.63	2.71	2.92	1.70	0.986	1.124	1.287	1.661	2.07
0.50	2.56	2.56	2.56	2.65	2.87	1.75	0.965	1.106	1.271	1.649	2.06
0.52	2.49	2.49	2.49	2.60	2.83	1.80	0.945	1.088	1.256	1.637	2.05
0.54	2.42	2.42	2.43	2.55	2.79	1.85	0.925	1.071	1.242	1.626	2.04
0.56	2.36	2.36	2.38	2.51	2.75	1.90	0.907	1.056	1.229	1.616	2.04
0.58	2.30	2.30	2.32	2.46	2.72	1.95	0.890	1.042	1.216	1.607	2.03
0.60	2.25	2.25	2.27	2.42	2.68	2.0	0.874	1.028	1.204	1.598	2.02
0.62	2.19	2.20	2.23	2.39	2.65	2.1	0.843	1.002	1.182	1.580	2.01
0.64	2.14	2.15	2.18	2.35	2.62	2.2	0.816	0.978	1.161	1.565	1.996
0.66	2.09	2.10	2.14	2.32	2.60	2.3	0.791	0.957	1.143	1.551	1.985
0.68	2.05	2.06	2.10	2.29	2.57	2.4	0.768	0.937	1.126	1.538	1.974
0.70	2.00	2.01	2.06	2.26	2.55	2.5	0.747	0.919	1.110	1.526	1.965
0.72	1.957	1.974	2.03	2.23	2.53	2.6	0.727	0.903	1.096	1.515	1.956
0.74	1.915	1.937	1.993	2.20	2.51	2.7	0.709	0.887	1.083	1.505	1.948
0.76	1.876	1.900	1.961	2.18	2.49	2.8	0.693	0.873	1.070	1.496	1.941
0.78	1.838	1.866	1.930	2.15	2.47	2.9	0.677	0.859	1.059	1.487	1.934
0.80	1.801	1.833	1.901	2.13	2.45	3.0	0.663	0.847	1.048	1.479	1.927

PROBLEMS

101. Find the discharge in a rectangular flume of new unplaned timber set on a grade of 2 feet per thousand feet. The flume has a bottom width of 8 feet and the depth of flow is 4 feet.

102. The following data were obtained in a determination of the effect of algae growth in a trapezoidal concrete-lined irrigation ditch. Bottom width 3 feet, side slopes 1 on 1, width of water surface 6 feet, $Q = 47$ c.f.s. Elevation at Sta. 91 + 00, 875.13, at Sta. 98 + 73, 863.47. Compute the value of n.

103. Uniform flow with a velocity of 6 feet per second is desired in a pyralin flume to be used in a laboratory investigation. If the section is circular, 1 foot in diameter, and is to flow half full, at what slope should it be constructed? What allowance for adjustment should be provided, to cover uncertainty in the roughness, if the length of the flume is 50 feet?

104. An earth channel lined with shot concrete has side slopes of 1.5 vertical to 1.0 horizontal, tangent to a 3-foot radius at the bottom, and is laid on a slope of 0.004. What will be the depth of uniform flow for a discharge of 250 c.f.s.?

105. What should be the bottom width of an earth canal with side slopes of 2 horizontal to 1 vertical and a uniform slope of 0.001, if it is to carry 2,000 c.f.s. at a depth of 8 feet?

CHAPTER II

BERNOULLI'S THEOREM APPLIED TO A FRICTIONLESS RECTANGULAR OPEN CHANNEL

Consider first, that water flowing along a horizontal rectangular channel with a high velocity is to be slowed up by means of a smooth gradual enlargement in the cross section. Assume that the flow is steady and that the effect of friction may be ignored. Bernoulli's theorem, then, states that if water moves in such a stream from one point to another at the same level, without loss of energy by friction or impact, the sum of the static head and the velocity head at the first point is equal to the sum of the same quantities at the second point.

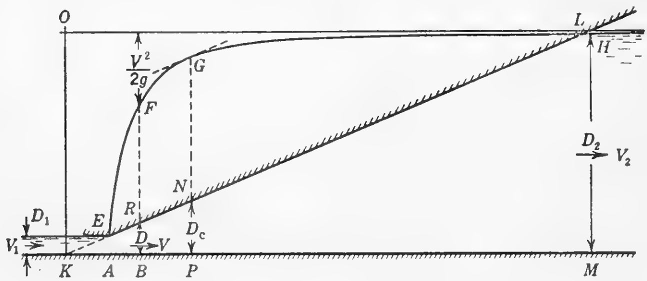


FIG. 201. Diagram Illustrating Flow in an Expanding Tube.

Constant width, level bottom, no friction or impact. The ordinates of the curve *EFGH* represent the static pressure on the bottom *AM*; the height of the curve *EFGH* above the straight line *EH* represents the static pressure on the top *EH*; and the distance of the curve *EFGH* below the line *OL* represents the head corresponding to the velocity within the tube. The critical conditions of flow exist at the section *NP*.

Let Fig. 201 represent such a condition. Suppose the original velocity is V_1 and depth D_1. If the channel is open at the top, the static head at the point K is D_1 and the velocity head is $V_1^2/2g$.

Suppose the channel to be of uniform width and the bottom level. In order to secure the gradual smooth enlargement of cross section, suppose a rigid top to be added to the channel as shown from E to H, rising along a straight line. We know by experience that if the angle is not made too great, the water will cling to the inclined plane and as the cross section increases the velocity will be smoothly and gradually reduced, as exemplified in the expanding tube of the Venturi meter.

At some point as B, where the depth has increased to D and the velocity has decreased to V, the velocity head will have diminished to $V^2/2g$.

Since the quantity of water flowing past each point is the same, let Q = volume of water per foot in width of the conduit, then

$$Q = V_1D_1 = VD \tag{201}$$

or

$$V = \frac{D_1V_1}{D} \tag{202}$$

By Bernoulli's theorem the static pressure at B then equals

$$D_1 + \frac{V_1^2}{2g} - \frac{V^2}{2g} = D_1 + \frac{V_1^2}{2g}\left(1 - \frac{D_1^2}{D^2}\right) \tag{203}$$

In general, this pressure at B would not equal D exactly, but would be something greater, as BF. By means of the above expression, the static pressure corresponding to each depth can be easily calculated for given conditions. This has been done for Fig. 201.

The resulting static pressures, when plotted above the base AM, give the curve $EFGH$. This curve, which crosses the surface of the water at E and H, indicates relations of great fundamental importance which should be carefully noted. At E the static pressure on the bottom is exactly that due to the depth of water AE. As soon as the velocity is reduced and part of the velocity head is thus converted into pressure, the pressure on the bottom is greater than that due to the depth of the water alone. Thus at P the pressure is that caused by a head GP. This means that at the point N there is a pressure against the upper bounding surface of the water equal to that caused by a head GN. If a small piezometer tube should be inserted through the upper surface at N, the water would rise in it to the level of G. This pressure tends to burst the cover off the conduit. The curve $EFGH$ is really the hydraulic grade line through the expanding section.

This interior bursting pressure against the upper surface begins with zero at E and increases, very rapidly at first, and then more slowly, until it reaches a maximum value such as GN at N. From this point onward it gradually decreases until it again reaches zero at H. The pressure on the bottom at M is again exactly that caused by the depth of the water.

As the velocity decreases between E and N, the corresponding diminution of velocity head is more than enough to raise the water along the rigid, slanting, upper boundary surface, and hence there is an accumulation of excess static pressure. As the velocity decreases from N to H,

the gradual conversion of velocity head into pressure is not sufficient to raise the water, and hence the previous accumulation of pressure is gradually drawn upon in raising the surface to higher and higher elevations.

If the whole apparatus is open to the air, the water at H would no longer follow the upper slanting surface, but would break loose and flow away with level surface from this point. If the air were excluded by suitable means beyond the point H, it might be possible to have the upper water surface continue to cling to the slanting face, in which case the water beyond H would be under a vacuum, increasing in intensity the farther the expansion of cross section is continued.

The pattern of the variation of pressure along the top EH is obviously reasonable from the following considerations. For a given small change in cross section, the change in velocity near E will be much greater than near H. Also, since the velocity head is proportional to the square of the velocity, for a given change in velocity, the change in velocity head will be greater as the velocity itself is greater. To illustrate, the difference between the squares of 101 and 100 is 201, though the difference between the squares of 11 and 10 is only 21. That is, a change of 1 in a velocity of 100 would have about 10 times as much effect on the velocity head as a change of 1 in a velocity of 10. For both of these reasons, then, the pressure changes most rapidly at E.

The cross section at which the upward pressure on the surface EH is a maximum, shown on Fig. 201 at NP, is of particular interest. At this place a tangent to the curve at G is parallel to the water surface at N, and the change of velocity head is just sufficient to produce the change in elevation of water surface, so that there is neither accumulation nor reduction of pressure. On account of this peculiar balance between velocity head and depth, this particular section may be said to mark a critical point or a condition of *critical flow*. We shall find hereafter that this critical point appears in numerous important relations. Let the subscript c denote values at the critical point. From the value of the static pressure on the bottom given by equation (203), it is easily proved by the differential calculus that

$$V_c = \sqrt{gD_c} = \sqrt[3]{gQ} \qquad [204]$$

and

$$D_c = \frac{V_c^2}{g} = \sqrt[3]{\frac{Q^2}{g}} \qquad [205]$$

From equation (205) it is evident that at the critical point the velocity head is one-half the depth. This leads to a convenient definition or

physical picture of the condition of critical flow. In any rectangular channel critical flow exists when the velocity head is one-half the depth of the moving stream. Thus in Fig. 201 at P the velocity head, represented by the vertical distance from G to the horizontal axis OL, is equal to one-half the depth, NP.

But a little further discussion of Fig. 201 is necessary. The curve $EFGH$, if extended according to its mathematical equation, has as asymptotes the straight lines OL and OK. The cover EH need not follow a straight line. Any smooth curve would do. The curve $EFGH$ would change correspondingly in shape, but would have the same properties as enumerated above, and would, for each depth of flow, be at the same distance above the top.

The depths at E and H always have a definite relation to each other because at these two points the sum of the depth and the velocity head is the same. Depths so related are called *alternate* depths.

Let D_1 = depth at E
D_2 = depth at H

Then

$$\frac{V_1^2}{2g} + D_1 = \frac{V_2^2}{2g} + D_2 \qquad [206]$$

Substituting for velocities their values in terms of Q, and transposing,

$$\frac{Q^2}{2gD_1^2} - \frac{Q^2}{2gD_2^2} = D_2 - D_1$$

$$\frac{Q^2}{2g} \cdot \frac{D_2^2 - D_1^2}{D_2^2 D_1^2} = D_2 - D_1$$

Dividing through by $D_2 - D_1$, and substituting D_c^3 for Q^2/g,

$$D_c^3 = \frac{2D_1^2 D_2^2}{D_1 + D_2} \qquad [207]$$

If the values of any two of the depths are given, the value of the third can readily be obtained. The equation is symmetrical in D_1 and D_2. When D_1 equals D_c, D_2 will also equal D_c. When D_1 is less than D_c, D_2 must be greater than D_c, and vice versa.

The method described above is not the only device by which theoretically the depth of water flowing in the rectangular channel may be varied. Suppose that instead of the rigid top applied to the conduit as shown in Fig. 201, the bottom be raised on a gradual ascent as shown in Fig. 202. The water will then flow up the incline as indicated in the figure. As the water rises from E, some of the initial kinetic energy will

be consumed in lifting the water. At first, when the water has a high velocity, sufficient energy can be supplied with but a slight decrease in velocity. Hence, on the lower part of the slope the depth will increase very slowly; but as greater heights are reached, it requires increasingly greater changes in velocity to supply the required energy, and therefore the depth, which varies inversely as the velocity, changes more rapidly.

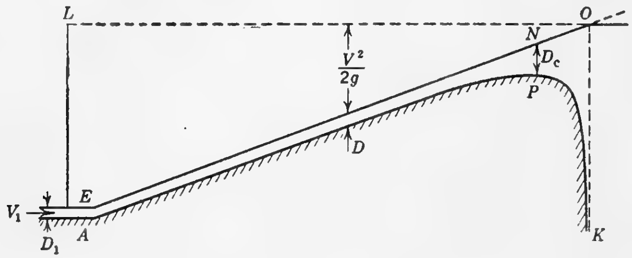


FIG. 202. Diagram Illustrating Flow in an Open Conduit of Constant Width.

No friction or impact. The profile of the bottom is that necessary to raise the surface of the water on the uniform slope *EO*; that is, the decrease in velocity head accompanying the increase in depth is just sufficient to raise the water along the line *EO*. The critical conditions of flow exist at the section *NP*.

When the water reaches the section *NP*, the depth must increase at the same rate as the surface rises. Hence, at this section, the bottom must be level and the depth is the critical depth previously described and shown at *NP* in Fig. 201. As stated before, at this critical point, the velocity head is one-half of D_c.

In order that the water surface may rise as the water flows beyond this section, the bottom must descend as indicated in Fig. 202. If the depth becomes infinite, so that the entire kinetic energy may be converted into static head, the surface may rise to *O* as a maximum limit.

The bottom need not follow exactly the curve shown. If it follows any smooth curve rising to the same maximum height *P*, the water surface will follow a corresponding curve having for each elevation the proper height above the bottom.

A third method might be used to raise the water by converting velocity head into static head. If the bottom of the conduit is kept level as in Fig. 201, but if the sides, instead of remaining parallel to give a uniform width to the channel, as in that diagram, are made to approach each other and then diverge, any desired rate of change of the velocity head may be secured. Figure 203 shows the variation in width necessary to secure a uniform slope to the water surface. The curves showing the variable width, if prolonged, would be asymptotic at the two ends to the straight lines *AG* and *OK*. Any other curves having the

same minimum opening *RS* would do as well. The only difference would be that the rising surface of the water would not be a plane but a curved surface.

So far we have been discussing a hypothetical condition of flow, and have assumed that there is no friction or loss of energy by impact. How far does this represent or approximate conditions as they may exist in actual flow of water? This is an important matter to determine, and fortunately our knowledge of hydraulic phenomena enables us to go a certain distance in answering this question. In the first place, it is to be remembered that of the three distinct methods for converting velocity

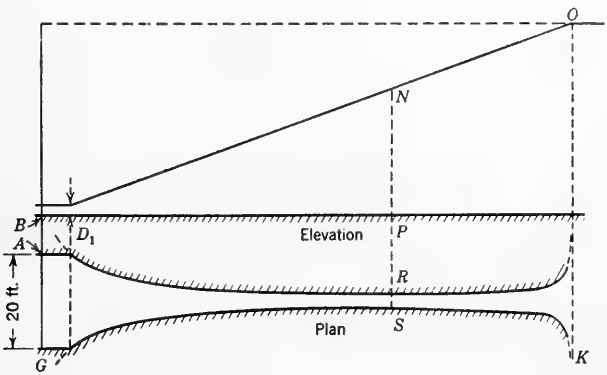


FIG. 203. Diagram Illustrating Flow in an Open Conduit of Variable Width.

Bottom level, no friction or impact. The variation in width is that necessary to raise the water surface on a uniform slope.

head into static head, described above and illustrated in Figs. 201, 202, and 203, any two, or all three, may operate together without conflict. Surface friction in absorbing energy, in one sense, is comparable to a rising slope of the conduit bottom, similar to that of the left-hand portion of Fig. 202. Hence, if the amount of friction is known, or is susceptible of estimate, it may be allowed for by combining the method of Fig. 202 with the other processes.

Impact is not so easily disposed of. In most hydraulic operations it is desired to avoid the occurrence of impact entirely. If it occurs, it is likely to introduce at once an element of extreme variability and uncertainty, not susceptible of accurate prediction or calculation except in certain cases. Hence the present question may be considered to be, with how much assurance, or with what certainty, may we assume that impact may be avoided in velocity transformations?

Impact does not occur in a flowing stream in which the velocity is

being accelerated, that is, in the condition in which pressure or elevation head is being converted into velocity head. Under the reverse condition, however, of converting velocity head into static head, there seems to be an inherent tendency for impact to occur, causing excessive turbulence and a corresponding decadence of kinetic energy into heat. Thus, although theoretically velocity head and static head are mutually interchangeable or convertible by a simple relation, in applying this relation we have to remember that the change from velocity head to static head can only be secured against obstacles, almost as though the water were animated by a desire to avoid the change, and would avoid it, if any way were open for it to do so. If the area of the stream increases too rapidly in the downstream direction, the velocity will not decrease uniformly. Part of the section will be filled with swiftly moving water, and part with slowly moving water. Impact will inevitably occur, and the energy lost in the resulting turbulence will prevent the static head developed from being as large as that computed, assuming only ordinary friction losses.

The change from static head to velocity head, on the other hand, occurs naturally and readily, with but little loss.

With this in mind, the methods shown in Figs. 201, 202, and 203 may be compared. Experience shows that by Fig. 201, it is comparatively easy to secure the results desired. The water confined on all sides cannot escape the transformation, and it is only necessary, for the desired result, that the angle of divergence be sufficiently small. By Fig. 202, the change is much more difficult. The water, being open at the top, has a direction of possible motion and a source of disturbance difficult to control. The place of particular danger is at N, the surface of the water when flowing at the critical depth. At this particular place the velocity head and elevation head are in a state of balance, the rate of increase of the latter being exactly equal to the rate of decrease of the former, so that the surface could take any other slope, even a vertical one, without energy having to be supplied or taken away. Being, then, in a state of unstable equilibrium, a very slight disturbance could seriously modify its motion.

Figure 203 shows the same condition of instability at the section of critical flow, RS. Because of the convergence and divergence of the flow, this method would probably be more difficult to adjust even than that of Fig. 202.

It is interesting to note that if the direction of flow is reversed, so that the velocity is increasing instead of decreasing, all three methods would work better. The first two methods, especially, could be made to follow the mathematical curve quite closely.

When the flow is from left to right, as shown in Figs. 201, 202, and 203, it is impossible for the depth to increase in the absence of friction or impact, except by means of some such agency as those illustrated. If the channel is of uniform width, with level bottom, open at the top, and frictionless, it is impossible for the depth of flow to increase except by a form of impact which will be discussed in the next chapter. A limited amount of variation is possible, however, in the immediate vicinity of the critical depth.

Flow at velocities higher than the critical is said to be rapid flow, shooting flow, or super-critical flow, while flow at velocities lower than the critical is said to be tranquil flow, streaming flow, or sub-critical flow. This criterion of flow, which, as we have seen, is based on a balance of the velocity head and the static head, was discovered by Belanger, and is often referred to as Belanger's critical flow. It should not be confused with other criteria of flow such as Reynolds', which is used to distinguish between turbulent and viscous flow.

PROBLEMS

201. What changes would result in Fig. 201 if Q was kept the same but D_1 made smaller? Larger?

202. Find the equation of the curve *EFGH* in Fig. 201.

203. Prove that the value of D_c in equation (207) is always between the values of D_1 and D_2.

204. Make a graph of equation (207). Use values of D_1/D_c as ordinates, and values of D_2/D_c as abscissas. For comparison, plot the rectangular hyperbola $D_1D_2 = D_c^2$ on the same graph.

CHAPTER III

THE STATIONARY HYDRAULIC JUMP IN CHANNELS OF RECTANGULAR CROSS SECTION

When a shallow stream moving with a high velocity impinges upon water of sufficient depth there is commonly produced a striking phenomenon which has been appropriately called the *hydraulic jump.* It consists of an abrupt rise in the surface in the region of impact between the rapidly moving stream and the more slowly moving wall of water, accompanied by a great tumbling of the commingling water, and the production of a white foamy condition through the moving mass. Under suitable conditions this hydraulic jump remains steadily in one position. The surface at the beginning of the abrupt rise is constantly falling against the oncoming stream moving at high velocity, and farther along in the jump, masses of water are continually boiling to the surface from greater depths. So much foam is produced that some time must elapse before it can all rise to the surface and the water again become clear. This phenomenon is constantly illustrated in the surf of the seashore.

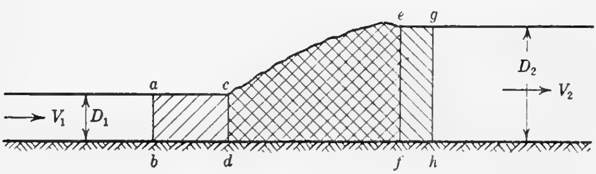


FIG. 301. Diagrammatic Longitudinal Section through a Hydraulic Jump.

Let *abfe*, Fig. 301, represent a mass of water moving through a hydraulic jump in a rectangular channel. In a short interval of time it is supposed to move to the position *cdhg*. The hydraulic jump has the following characteristics:

(*a*) The water entering at *ab* has a nearly uniform steady high velocity and is transparent.

(*b*) The water leaving at *gh* has a fairly uniform but relatively low velocity and is transparent.

(*c*) Between *c* and *e* the surface rises rapidly and is much disturbed by spraying and spattering. Much of the surface water appears to be moving down the slope. The whole mass is full of small bubbles, and is violently agitated by the extreme turbulence of the flow. The milky

condition of the water reduces its specific gravity and accordingly the surface at the top of the rise is above the normal level, but as soon as all the air bubbles reach the surface, so that the water is again transparent, the surface becomes comparatively smooth and level.

The moving mass of water loses much momentum in passing from the position *abfe* to the position *cdhg*. According to Newton's second law of motion, the rate of loss of momentum must be equal to the unbalanced force acting on the moving mass to retard its motion.

Against the face *ab* is the static pressure of the water acting towards the right. Opposed to this force are the static pressure acting against the face *ef* and the surface friction along the bottom *bf*. The latter is small and may be neglected.

Let D_1 = depth of stream entering hydraulic jump,
V_1 = velocity of same stream,
D_2 = depth of stream leaving hydraulic jump,
V_2 = velocity of same stream,
D_c = depth of the stream at critical flow,
V_c = velocity at critical flow, $V_c^2 = gD_c$,
Q = quantity in second-feet of flow per unit width of stream,

$$Q = V_1D_1 = V_2D_2 = V_cD_c = \sqrt{gD_c^3}$$

$$\text{Mass of water flowing per second} = \frac{wQ}{g},$$

$$\text{Change of velocity} = V_1 - V_2,$$

$$\text{Change of momentum per second} = \frac{wQ}{g}(V_1 - V_2),$$

$$= \frac{wQ}{g}\left(V_1 - V_1\frac{D_1}{D_2}\right) = \frac{wQV_1}{gD_2}(D_2 - D_1),$$

$$\text{Static pressure acting on face } ab = \frac{wD_1^2}{2},$$

$$\text{Static pressure acting on face } ef = \frac{wD_2^2}{2}.$$

Therefore,

$$\frac{wQV_1}{gD_2}(D_2 - D_1) = \frac{w}{2}(D_2^2 - D_1^2)$$

Dividing both members by $w(D_2 - D_1)$

$$\frac{QV_1}{gD_2} = \frac{D_1 + D_2}{2} \qquad [301]$$

from which may be obtained

$$D_2^2 + D_2D_1 = \frac{2QV_1}{g} \qquad [302]$$

or, after substituting V_1D_1 for Q,

$$D_2 = -\frac{D_1}{2} \pm \sqrt{\frac{2V_1^2D_1}{g} + \frac{D_1^2}{4}} \qquad [303]$$

$$D_2 = -\frac{D_1}{2} \pm \sqrt{\frac{2Q^2}{gD_1} + \frac{D_1^2}{4}}$$

By substituting Q/D_1 for V_1 in equation (302) we get

$$D_1D_2\frac{(D_1 + D_2)}{2} = \frac{Q^2}{g} = D_c^3 \qquad [304]$$

Equation (304) shows that if $D_1 = D_c$, D_2 also equals D_c. The equation is symmetrical in D_1 and D_2. If D_1 is less than D_c, D_2 must be correspondingly greater than D_c, and vice versa. There seems to be no physical phenomenon corresponding to a reversal of the jump in direction. Therefore, there can be no jump unless D_1 is less than D_c, and the jump, when it occurs, always takes place across the critical depth.[1] As already illustrated in Fig. 202, water can flow at depths less than the critical depth without necessarily forming a jump. This will be more fully discussed later in connection with backwater curves, where the conditions determining the formation of a jump will be more completely stated.

The equation of the hydraulic jump can be written in a great many different ways, in terms of the different possible fundamental and derived variables. If any two of the four simplest variables, D_1, D_2, V_1, and V_2, are given, the values of the others can be determined. Figure 502, page 57, permits a direct graphical solution.

The above demonstration rests upon a law of mechanics as well established as any law of nature, as well proved, for example, as the law of gravitation, and there can be no question of the validity of the results. In the hydraulic jump there is continuous violent impact, and by means of the resulting turbulence a part of the kinetic energy is converted into heat.

For given values of D_1 and D_c, the value of D_2 given by equation (304) will always be less than that given by equation (207). This is because the change in depths represented by equation (207) occurs without loss

[1] This statement applies only to the stationary jump.

of energy, whereas the change represented by equation (304) always involves loss of energy. The loss of energy approaches zero as D_1 approaches D_2, or as the height of the jump becomes infinitesimal. The depths before and after the jump approach the critical depth, which is the depth at which small fluctuations in depth may occur freely.

Consider the different possible depths at which a given quantity of water can flow in a rectangular frictionless open channel with parallel sides, for different values of the total head. The constant width of the channel may be assumed to be unity, so that

$$Q = VD$$

Denoting the total head by H,

$$H = D + \frac{V^2}{2g}$$

Eliminating V between the two equations,

$$D^2(H - D) = \frac{Q^2}{2g} = \text{a constant} \qquad [305]$$

This relation between the depth of flow and the total head is plotted in Fig. 302. Values of abscissas and ordinates are plotted in terms of the critical depth. This diagram shows (1) that the total head is a minimum when it is two-thirds depth and one-third velocity head. When at this

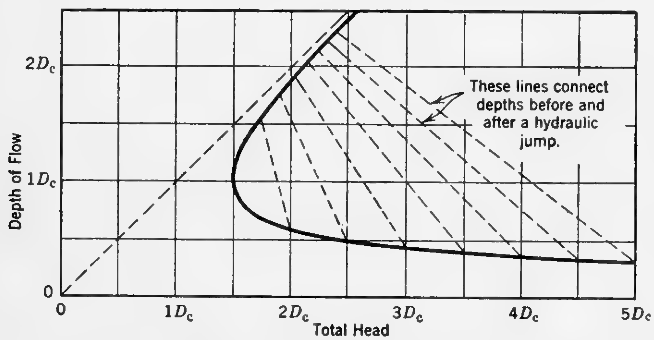


FIG. 302. Depths of Flow in a Rectangular Channel, for Constant Discharge.

stage, the flow is critical, and the depth may change appreciably with comparatively no change in total head. (2) When the flow is not critical, two depths of flow are possible at any given total head, one greater, and one less, than the critical depth. These are the alternate depths illustrated by Fig. 201. When water is flowing freely at either

of these depths, a small change of depth cannot occur without a corresponding change in total head. (Flow as in Figs. 201, 202, or 203 is excluded from consideration, for Fig. 302 represents flow in a parallel-sided, horizontal, rectangular channel, with free surface.)

The dotted lines on Fig. 302 connect values of D_1 and D_2 before and after a hydraulic jump. The horizontal distance between the ends of one of the dotted lines represents the loss of total head in passing through the jump.

The alternate depths, for which the total head is the same, should not be confused with the depths before and after a jump, for which the total head is not the same, except for very low jumps. Depths before and after a jump have been called conjugate depths, to distinguish them from alternate depths, but the writers prefer a term suggested by M. F. Thorne, *sequent depths*. The word conjugate is almost synonymous with alternate, and implies reversibility. Sequent, on the other hand, carries the idea of a definite, irreversible, order of sequence.

Near the critical depth there is very little loss of energy in the jump, compared with its height. The standing waves which occur easily, when the flow is critical, might be considered to be small jumps. This is another way of explaining the instability of the water surface for depths of flow near the critical depth.

Experimental verification of the jump theory. The first measurements of the hydraulic jump phenomenon apparently preceded the development of the correct theory. It should be remembered, in comparing the results of experiments with the theory, that many of the experiments were made in small channels, or in channels with artificially roughened bottoms, in which case the effect of friction prevents exact agreement. Figure 303, adapted from Riegel and Beebe, shows a dimensionless plot of observed data, the momentum formula, and the formula which would be correct if Bernoulli's theorem applied to the hydraulic jump; that is, if there were no loss of energy through the jump. It is seen that Gibson's experiments verify the law most closely. This is to be expected, for his channel was wide, and most nearly approached the ideal rectangular frictionless channel. For very low jumps, the points scatter, some even showing an apparent gain of energy. This illustrates the difficulty of making precise measurements in the immediate neighborhood of the critical depth.

Length of the hydraulic jump. It has been shown that if water is flowing in a smooth, uniform, rectangular open channel with level bottom, it is impossible for the flow to change to a different depth in accordance with Bernoulli's theorem, unless the flow is at critical stage, when small fluctuations only may occur. It has been shown subse-

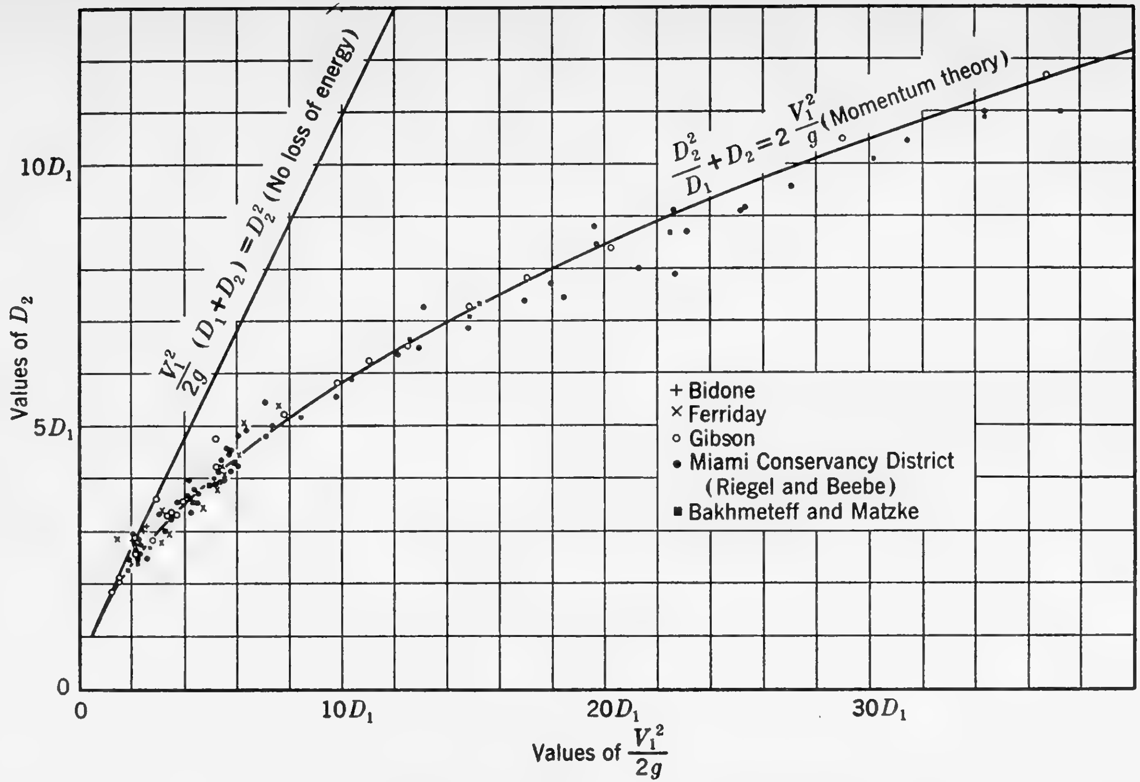


Fig. 303. Comparison of Momentum Formula with Experimental Data.

Some of the Miami Conservancy District tests were run in a channel 10 feet wide. Gibson's channel was 3 feet wide; the others were narrower. The greatest value of D_2 observed in any of the tests was 2.32 feet.

quently that, under the theory of inelastic impact, the depth can change abruptly from a value less than the critical depth to a particular related value greater than the critical depth. The change in the jump from the initial velocity to the final velocity, however, is very evidently not instantaneous. At points between the cross sections where the initial and final velocities occur, the average velocity must have intermediate values. At first glance this would seem to contravene the theory of the jump, which seems to preclude any value for the average velocity after the initial value, except the calculated final one.

A reason for this may be found if we examine conditions within the jump. Except in very small jumps in which surface tension prevents their entrance, the water is filled with thousands of minute bubbles. These rise to the surface quickly, leaving clear water. They undoubtedly assist in the dissipation of energy and in smoothing out the violent velocity variations. On the steep forward surface of the jump, a roller forms, tumbling in erratic manner against the rapidly flowing sheet below. The admixture of air reduces the specific gravity of the liquid, and this increases the static pressure on a vertical section. By finding how much the static pressure needs to be increased in order that the law of momentum may be satisfied at every intermediate section, a hypothetical profile through the jump may be computed.[2] Unfortunately this profile does not fit observed data; it is too high. It should not be expected to fit, for the assumption is made that the velocity is uniform throughout each vertical section. Owing to the presence of the roller, the velocity distribution is far from uniform.

Another hypothetical profile may be computed by assuming that the roller has no resultant downstream momentum, but that it supplies the necessary pressure on top of the expanding jet to preserve continuity in the pressure-plus-momentum relations.[3] This profile does not fit the observed data; it is too low.

No mathematical analysis of conditions within the jump which will give results that consistently agree with experiment is known to the authors. Apparently the answer to this difficult problem must await laboratory investigation. A few investigations have been made, especially directed toward determining the length of the jump. There is a lack of agreement among the different investigators, and until this has been resolved by further experimentation, it seems unwise to accept any

[2] Miami Conservancy District *Technical Reports*, Part III, p. 28 et seq.

[3] "The Hydraulic Jump and its Top Roll and the Discharge of Sluice Gates," by Kazimierz Woycicki, Chapters III & IV, translated by I. B. Hosig, Bureau of Reclamation, *Technical Memorandum* 435.

of the formulas proposed.[4] In most of the experiments the length of the jump was found to be between $4\frac{1}{2}$ and 7 times its height.

Location of the hydraulic jump. It is important to be able to determine just where, along the length of the channel, the hydraulic jump will form. Although this cannot be done with a great deal of precision, the location of the jump can nevertheless be predicted within certain limits, depending upon the accuracy with which the friction losses and length of jump can be estimated. The method as used for rectangular

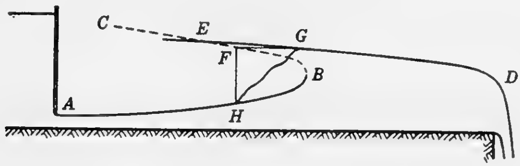


FIG. 304. Location of the Hydraulic Jump.
(The vertical scale is distorted.)

channels is illustrated in Fig. 304. The water surface curves for the flow approaching the jump AB and leaving the jump ED are computed by one of the backwater-curve methods explained in Chapters VI, VII, or IX. The curve CB is a plot of depths sequent to the depths of curve AB. The line FG is a horizontal line of length equal to the estimated length of the jump, connecting the curves CB and ED. Since the depth at G is sequent to the depth at H, and approximately the correct distance downstream, the jump will form at the location HG. In this case the jump forms far downstream from the point E, where the curve of sequent depths intersects the tailwater curve, and where the jump might be expected to form, were its length not taken into account. The necessity of considering the length of the jump was pointed out by J. W. Trahern.[5]

Loss of energy in the hydraulic jump. The energy loss per unit of space occupied is probably greater in the hydraulic jump than in any other hydraulic phenomenon. The greater portion of the energy loss occurs under the steep part of the jump, and comparatively little energy goes into unbalanced velocity distribution, to be dissipated downstream from the jump. This is in marked contrast with such a phenomenon as the flow around a bend, where there is little actual energy loss in the

[4] "The Hydraulic Jump in Terms of Dynamic Similarity," by B. A. Bakhmeteff and A. E. Matzke, *Transactions A.S.C.E.*, Vol. 101, 1936. This paper and its discussions contain summaries of, or references to, most of the work that has been done in this field.

[5] "Location of the Hydraulic Jump," by J. W. Trahern, *Western Construction News and Highways Builder*, Oct. 25, 1932, p. 608.

bend itself, the water being given an abnormal velocity distribution which persists for some distance downstream before its energy is dissipated.

If the hydraulic jump is "drowned out," that is, if the tailwater is raised so high that the jump no longer forms, the high velocity sheet will continue in an erratic and fluctuating manner for a considerable distance under and through the slower water. Conditions then are more like those in the bend. The energy loss is not concentrated in a small space, but is spread out through a large volume.

The energy loss through a hydraulic jump which is functioning properly may be evaluated theoretically. By Bernoulli's theorem the loss of head (including that due to friction from the bottom and sides of the channel, which is small) is

$$H' = \frac{V_1^2}{2g} + D_1 - \frac{V_2^2}{2g} - D_2$$

Introducing equation (304) which was derived from the momentum relation, and the law of continuity $Q = V_1D_1 = V_2D_2$ which applies because the flow is steady, the loss of head through the jump may be expressed in terms of velocities

$$H' = \frac{(V_1 - V_2)^3}{2g(V_1 + V_2)} \qquad [306]$$

or depths

$$H' = \frac{(D_2 - D_1)^3}{4D_2D_1} = (D_1 + D_2)\left(\frac{D_2 - D_1}{2D_c}\right)^3 \qquad [307]$$

It is seen that the loss becomes greater as the height of the jump increases, and approaches zero as the height of the jump approaches zero.

Uses of the hydraulic jump. Perhaps the earliest practical use of the hydraulic jump was as a "head increaser." Water to be discharged at as low an elevation as possible is caused to flow into the passing stream at a point where the velocity is high and the depth low, upstream from a hydraulic jump.[6]

The hydraulic jump frequently forms close to the toe of overflow dams. If it forms at all stages, and remains close to the dam, it is a particularly efficient means of destroying the excess energy of the overflowing water, and preventing the dam from being undermined and washed out. Unless the dam and apron have been carefully designed to insure the

[6] Among the unpublished papers of the late D. L. Yarnell is found reference to the use of the hydraulic jump in this manner, described, he said, in *Del Moto E Misura Dell' Acqua*, by Leonardo da Vinci (Bologna, 1828).

formation of the jump at all stages, it will be a coincidence if the jump forms over any considerable range of discharges.

The hydraulic jump may be used for mixing liquids, and as an aeration device. Its use for the former purpose was patented by J. W. Ellms.

The hydraulic jump as an energy dissipator. In order that the hydraulic jump may function ideally as a dissipator of the excess energy of water flowing over dams, it is necessary for the elevation of the water surface after the jump to coincide with the normal tailwater elevation, for every discharge. If the tailwater is too low, the high velocity stream will continue on downstream. If it is too high, the jump will be drowned out. In either case dangerous erosion is likely to occur for a considerable distance below the dam. The ideal condition is to have the rating curve for elevations after the jump coincide exactly with the tailwater rating curve.[7] It is best to accomplish this as nearly as possible in the design, leaving only minor adjustments to be made at the time of the model verification. These adjustments are customarily made by trial-and-error placing of artificial roughnesses, blocks, or piers, as required for satisfactory operation of the model. In high-head developments it is best to avoid blocks or dentates of any kind, for they erode rapidly, and make maintenance charges high. If the head is low, artificial roughnesses may be used more safely. This is fortunate, for it is in low-head developments that adjustment to secure satisfactory operation of the jump at all stages is most difficult to obtain. If a perfect jump can be secured at all stages, over a smooth apron, the energy will be dissipated without appreciable wear, and upkeep will be negligible.

One possible source of damage, which must be guarded against in design and construction of the apron, is that the full hydrostatic pressure of the depth after the jump may gain access to the under side of the apron, causing it to be lifted up under the thin sheet of water before the jump.

The position of the "jump-height" rating curve, an example of which is shown in Fig. 305, may be shifted by the following changes in the design:

(1) Changing the crest length. Lengthening the crest will lower the rating curve AB; shortening it will raise AB.

(2) Changing the elevation of the apron. Lowering the apron will lower the curve AB.

(3) Sloping the apron. The apron should not be given too great a slope, for the dissipation of energy becomes less efficient as the slope is

[7] See "Protection Against Scour Below Overfall Dams," by E. W. Lane and W. F. Bingham, *Engineering News-Record*, Mar. 14, 1935, pp. 373–378.

increased. Sloping the apron has the effect of giving satisfactory operation over an elongated area, in place of the narrow line AB, and greatly facilitates fitting the natural tailwater rating curve in certain cases.

Laboratory research of a fundamental nature is needed to test means of adapting the hydraulic jump energy dissipator for satisfactory operation over a wider range of conditions than is now considered possible.

The hydraulic jump in a channel with sloping bottom. In the preceding section, reference was made to the expedient of sloping the apron on which the jump forms in order to gain satisfactory operation over a wider range of tailwater conditions. As long as the slope is flat, this procedure is satisfactory, but when the slope is too steep, the high velocity water shoots out under the pool with little localized impact, making dangerous erosion likely. Laboratory experiments seem to indicate that the limit between the two types of action occurs at a slope of 5 or 6 horizontal to 1 vertical.

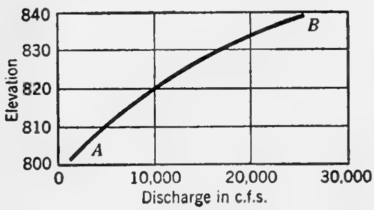


FIG. 305. Typical Rating Curve for Depths Downstream from a Hydraulic Jump.

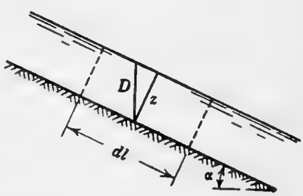


FIG. 306. Element of a Stream on a Steep Slope.

The mathematical analysis which gives results agreeing so precisely with observed data, in the case of the jump on a horizontal or gently sloping floor, leads to no such useful result for the jump on a steeply sloping floor. The results of an analysis are instructive, however.

Consider first the pressure against the slope, for a stream of vertical depth D and thickness z perpendicular to the slope. The weight of an elementary volume of unit width and length dl is $wzdl$. The depth is assumed not to vary appreciably within the length dl, so that the forces on the ends and sides of the elementary volume are equal in magnitude and opposite in direction. The unit pressure against the plane is therefore

$$wz \cos \alpha = wD \cos^2 \alpha$$

The factor $\cos^2 \alpha$ is of course nearly equal to unity unless the slope is great. The difference is less than 1 per cent until α is nearly 6 degrees, a slope of about 1 in 10.

A longitudinal section through a hydraulic jump on a sloping floor is shown in Fig. 307. The side walls of the flume in which the jump forms are assumed to be vertical and parallel. The unbalanced force on

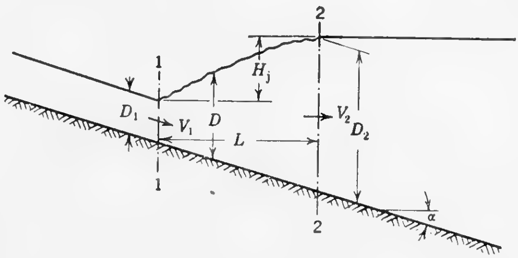


FIG. 307. Diagrammatic Longitudinal Section through a Hydraulic Jump on a Sloping Floor.

the water between sections 1–1 and 2–2 acts horizontally toward the left, and is approximately equal to

$$\frac{w(H_j + D_1 + L\tan\alpha)^2}{2} - \frac{wD_1{}^2\cos^2\alpha}{2} - \int_{1-1}^{2-2} wDdA\sin\alpha$$

where dA is an element of the area of the bottom, at depth D. The time rate of change of momentum is

$$\frac{wQ}{g}\left(V_1\cos\alpha - \frac{Q}{H_j + D_1 + L\tan\alpha}\right)$$

assuming that the velocity at section 2–2 is uniform and approximately horizontal in direction. These two quantities could be equated, and the resulting equation solved for H_j. It is unnecessary to do so, however, for it is seen that H_j depends not only upon D, Q, and α, but also upon the shape of the profile and the length of the jump. The latter factors, which are unknown even for the jump on a level floor, and which must certainly vary with α, exert a large influence upon the value of H_j. Attempts to avoid this difficulty by considering the components of forces and momenta along the slope are equally fruitless, for then the component of the weight of the water between the initial and final sections must be included. Without knowing the surface profile through the jump this factor cannot be estimated.

It is somewhat disappointing not to be able to obtain a formula as simple as the well-verified formula for the height of the jump in a level channel, but it is questionable whether such a formula would be of much use if it could be derived. We know by experience that when the jump

forms on a very steep slope, there is little depression of the water surface near where the jet plunges. The position of the beginning of the jump is fixed within narrow limits by the intersection of the downstream pool surface, which is very nearly level, and the surface of the swiftly flowing stream on the slope. If the jump forms on a comparatively flat slope, on the other hand, it does not start at this intersection, but starts some distance downstream, depending upon the height of the jump. Thus it is for flat slopes that we need most to know the height of the jump, and fortunately, it is for these cases that the theory of the jump on a level floor will apply with sufficient accuracy.

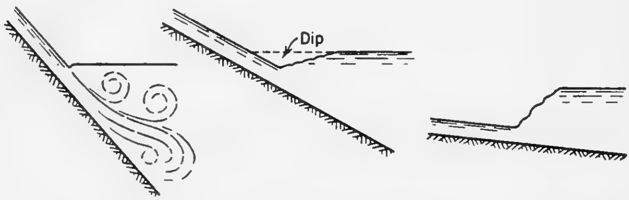


FIG. 308. The Hydraulic Jump in Channels Having Different Slopes.

The level-floor jump formulas will apply with satisfactory accuracy for slopes up to 10 or 15 per cent. Above this is an intermediate range about which more information is needed. The information which is needed is not so much the height of the jump as its length. For slopes above 10 or 15 per cent, the downstream water surface will be nearly level, and the length of the jump may be measured from the intersection of the level of the downstream pool with the upstream water surface. The actual amount of dip of the water surface below the pool level is of little practical importance, for side walls would have to be built full height so as not to be overflowed at low discharges with high tailwater.

Experimental investigation of the length of the jump in sloping channels is as urgently needed as investigation of the length of the jump in horizontal channels. For steep slopes, the " jump " action is very imperfect, and erratic fluctuations of the high velocity jet continue far out into the pool. We need to know at what slope normal jump action ceases and this plunging flow begins. Because of the unknown effect of viscosity, surface tension, etc., experimental studies should be made on nearly full-size models.

PROBLEMS

301. Fill in the blank spaces in the following table, in which each horizontal line represents a hydraulic jump in a frictionless rectangular channel with horizontal bottom.

D_1	V_1	D_2	V_2	H'	L_j
ft.	ft./sec.	ft.	ft./sec.	loss of head	length of jump
1.0	16				
		7	4		
0.8		4.5			
	8		2		

302. If the head on the smooth, rounded crest of a dam is 3 feet, what tailwater elevation will be necessary to insure the formation of a hydraulic jump on a level apron 20 feet below the crest of the dam? If this tailwater elevation should happen not to coincide with the natural water-surface elevation of the stream, what changes in the spillway might be made to raise or lower it?

CHAPTER IV

CRITERIA OF FLOW AND THE HYDRAULIC JUMP IN CHANNELS OF NON-RECTANGULAR CROSS SECTION

Critical depth. In Chapter II the critical depth was found to be that depth of flow, in a rectangular channel with level bottom, at which the rate of change of velocity head is just equal to the rate of change of depth. (Though these two rates are numerically equal when the flow is critical, it should be noted that they are opposite in sign.) For channels of general prismatic shape, this fundamental definition still applies, but a more convenient criterion is that which states that the flow is critical when the velocity head is equal to one-half the *average* depth. To prove this, let

D = depth, measured from the bottom, and

T = width of the section at the water surface, or top width.

Then, since the flow is steady,

$$V = \frac{Q}{A} \qquad dV = -Q\frac{dA}{A^2} \qquad \frac{dV}{dA} = -\frac{Q}{A^2} = -\frac{V}{A}$$

But $dA = TdD$, by geometry, so that

$$\frac{dV}{dD} = -\frac{TV}{A}$$

At the condition of critical flow

$$\frac{d}{dD}\left(\frac{V^2}{2g}\right) = -1 \qquad \frac{dV}{dD} = -\frac{g}{V}$$

Eliminating dV/dD, we find that at the critical stage

$$\frac{V^2}{2g} = \frac{A}{2T} = \frac{D_{\text{av.}}}{2} \qquad [401]$$

Two other definitions of critical flow are more significant, though not so convenient for computations. The first states that for a given total head above the bottom of the channel, the discharge is a maximum; the second that for a given discharge, the total head above the bottom of the channel is a minimum. These definitions may be proved simul-

taneously. For both, the derivative of the discharge with respect to the velocity should be zero. In the first definition the discharge is to be a maximum, and in the second it is to be constant.

$$\frac{d}{dV}(Q) = 0 \qquad \frac{d}{dV}(AV) = 0 \qquad \frac{dA}{dV} = \frac{-A}{V}$$

The derivative of the total head above the bottom of the channel, with respect to the velocity, is likewise equal to zero. In the first case it is to be constant, and in the second it is to be a minimum.

$$\frac{d}{dV}\left(\frac{V^2}{2g} + D\right) = 0 \qquad \frac{dD}{dV} = \frac{-V}{g}$$

The values of dA/dV and dD/dV are the same as in the proof of the preceding definition. Substitution and elimination of dD/dV will therefore give equation (401), as before, so that the flow must be critical.

Alternate depths. For a given discharge and total head only two different depths of flow are possible, except when the flow is critical, when the depth may vary continuously over a limited range. The

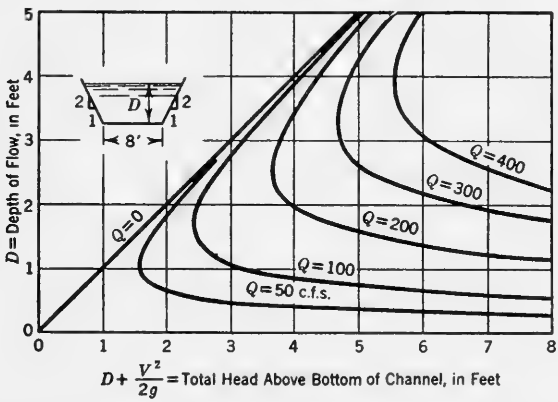


FIG. 401. Total Head as a Function of Discharge and Depth of Flow, in a Typical Trapezoidal Channel.

relation between these alternate depths is easily obtained for channels with cross sections in which the area is a simple function of the depth. Equation (207) gives the relation for rectangular channels, and similar expressions may be derived for triangular and trapezoidal cross sections. Alternate depths in channels having more complicated cross sections, however, are best obtained by a semi-graphical method. The procedure is indirect. A number of depths are first selected, covering the range

within which the desired results may reasonably be expected to fall. Next, the total head is computed for each of these depths, corresponding to the given discharge and the geometrical properties of the cross section. A curve is then plotted, using the depths as ordinates and the values of total head as abscissas. Alternate depths are selected by inspection of the curve, being the pairs of depths for which the total head is the same. A number of such curves, plotted for different discharges, are shown in Fig. 401. Alternate depths for discharges between those plotted may be obtained by interpolation. When one complete Q-curve has been plotted, others may be quickly added if advantage is taken of the fact that horizontal distances from the $Q = 0$ line are proportional to the square of the value of Q.

The computational procedure to be employed will depend upon the circumstances. Thus, if only the critical depth is needed for one value of discharge in a channel, the cut-and-try procedure based upon the criterion

$$\frac{V_c^2}{2g} = \frac{D_{\text{av.}}}{2}$$

will suffice, or tables may be used if these are available for the cross section. If a number of alternate depths are to be computed in addition, computation of the critical depth by the criterion may be omitted, for the critical depth will be determined by the minimum (total head) point of the curve used to determine the pairs of alternate depths.

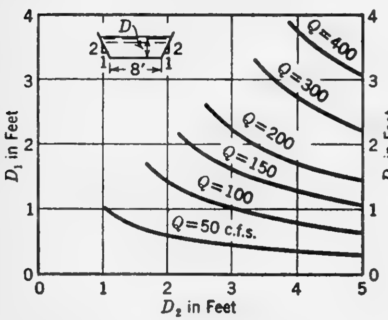


FIG. 402. Alternate Depths in a Typical Trapezoidal Channel.

When only a single depth, alternate to a given depth, is needed, the entire curve need not be computed. The alternate depth may be computed by a cut-and-try procedure, or a small portion of the curve may be computed.

Again, it may be necessary to compute a very large number of alternate depths in a given channel. Selection of the values from the diagram similar to Fig. 401 becomes burdensome, and it is desirable to prepare another type of diagram from which the alternate depths may be read directly. Such a diagram is shown in Fig. 402, which has been prepared for the same data shown in Fig. 401.

The variety of problems of this type that may arise is great. In

general, it is worth while to select the method of solution carefully, and if a large number of solutions has to be made, spend some time in planning a method that will minimize the work.

Table 401 will facilitate the solution of problems involving the critical depth and alternate depths in channels of circular cross section.

Illustrative Examples

Find the critical depth for a discharge of 75 c.f.s. flowing in a trapezoidal channel which has a bottom width of 10 feet and side slopes of 1 vertical to 1 horizontal.

The problem will be solved by the cut-and-try procedure. The successive trials are arranged in columns.

Depth, D	1	1.2	1.15	1.16
Area, $A = 10D + D^2$	11	13.44	12.82	12.95
Surface width, $T = 10 + 2D$	12	12.4	12.3	12.32
Average depth, $D_{av.} = \dfrac{A}{T}$	0.917	1.083	1.042	1.050
Average velocity, $V = \dfrac{Q}{A} = \dfrac{75}{A}$	6.82	5.58	5.85	5.79
Velocity head	0.723	0.484	0.532	0.521
Velocity head ÷ average depth	0.788	0.447	0.511	0.496

The critical depth is 1.16 feet, to the nearest hundredth, for with flow at that depth, the ratio of the velocity head to the average depth is most nearly one-half.

Find the critical depth for a discharge of 25 c.f.s. in a circular conduit 4 feet in diameter.

Table 401 lists values of $Q_c/D^{5/2}$ corresponding to values of d/D, where d is the depth of flow and D the diameter of the section.

$$\frac{Q_c}{D^{5/2}} = \frac{25}{32} = 0.781$$

By interpolation in the table, $d/D = 0.369$, so that the desired value of the critical depth is $0.369 \times 4 = 1.47$ feet.

PROBLEMS

401. (*a*) What is the critical depth for a discharge of 400 c.f.s. in a trapezoidal channel with bottom width of 20 feet and side slopes of 1 vertical to 2 horizontal?

(*b*) What is the critical depth in the same channel, when the total head is 15 feet?

(*c*) What is the depth alternate to a depth of $\frac{1}{2}$ foot, if the discharge is 400 c.f.s.?

402. (*a*) What is the critical depth for a discharge of 20 c.f.s. in a circular pipe culvert 3 feet in diameter?

(*b*) What is the depth alternate to a depth of 1 foot for the same discharge in the same channel?

TABLE 401

Hydraulic Properties of Circular Sections or Pipes Flowing Partly Full

d = depth of flow
D = diameter of pipe
r = radius of pipe
A = area of flow
W = surface width
P = total hydrostatic pressure
w = unit weight of water
Q_c = discharge when the flow is critical (Q_c in c.f.s., D and d in ft., acceleration of gravity assumed equal to 32.16 ft. per sec. per sec.)

$\frac{d}{D}$	$\frac{A}{D^2}$	$\frac{W}{D}$	$\frac{P}{wr^3}$	$\frac{Q_c}{D^{5/2}}$	$\frac{d}{D}$	$\frac{A}{D^2}$	$\frac{W}{D}$	$\frac{P}{wr^3}$	$\frac{Q_c}{D^{5/2}}$
0.01	0.0013	0.1990	0.0001	0.0006	0.51	0.4027	0.9998	0.6985	1.4494
0.02	0.0037	0.2800	0.0002	0.0025	0.52	0.4127	0.9992	0.7311	1.5041
0.03	0.0069	0.3412	0.0007	0.0055	0.53	0.4227	0.9982	0.7645	1 5598
0.04	0.0105	0.3919	0.0014	0.0098	0.54	0.4327	0.9968	0.7987	1.6166
0.05	0.0147	0.4359	0.0024	0.0153	0.55	0.4426	0.9950	0.8337	1.6741
0.06	0.0192	0.4750	0.0037	0.0220	0.56	0.4526	0.9928	0.8696	1.7328
0.07	0.0242	0.5103	0.0054	0.0298	0.57	0.4625	0.9902	0.9062	1.7924
0.08	0.0294	0.5426	0.0076	0.0389	0.58	0.4724	0.9871	0.9435	1.8531
0.09	0.0350	0.5724	0.0102	0.0491	0.59	0.4822	0.9837	0.9817	1.9147
0.10	0.0409	0.6000	0.0132	0.0605	0.60	0.4920	0.9798	1.0207	1.9773
0.11	0.0470	0.6258	0.0167	0.0731	0.61	0.5018	0.9755	1.0604	2.0410
0.12	0.0534	0.6499	0.0207	0.0868	0.62	0.5115	0.9708	1.1010	2.1058
0.13	0.0600	0.6726	0.0253	0.1016	0.63	0.5212	0.9656	1.1423	2.1717
0.14	0.0668	0.6940	0.0303	0.1176	0.64	0.5308	0.9600	1.1844	2.2386
0.15	0.0739	0.7142	0.0360	0.1347	0.65	0.5404	0.9539	1.2272	2.3068
0.16	0.0811	0.7332	0.0422	0.1530	0.66	0.5499	0.9474	1.2708	2.3760
0.17	0.0885	0.7513	0.0489	0.1724	0.67	0.5594	0.9404	1.3152	2.4465
0.18	0.0961	0.7684	0.0563	0.1928	0.68	0.5687	0.9330	1.3603	2.5182
0.19	0.1039	0.7846	0.0643	0.2144	0.69	0.5780	0.9250	1.4062	2.5912
0.20	0.1118	0.8000	0.0730	0.2371	0.70	0.5872	0.9165	1.4528	2.6656
0.21	0.1199	0.8146	0.0822	0.2609	0.71	0.5964	0.9075	1.5002	2.7416
0.22	0.1281	0.8285	0.0921	0.2857	0.72	0.6054	0.8980	1.5482	2.8188
0.23	0.1365	0.8417	0.1027	0.3116	0.73	0.6143	0.8879	1.5970	2.8977
0.24	0.1449	0.8542	0.1140	0.3386	0.74	0.6231	0.8773	1.6465	2.9783
0.25	0.1535	0.8660	0.1259	0.3667	0.75	0.6319	0.8660	1.6968	3.0606
0.26	0.1623	0.8773	0.1386	0.3957	0.76	0.6405	0.8542	1.7476	3.1450
0.27	0.1711	0.8879	0.1519	0.4259	0.77	0.6489	0.8417	1.7992	3.2314
0.28	0.1800	0.8980	0.1659	0.4571	0.78	0.6573	0.8285	1.8514	3.3200
0.29	0.1890	0.9075	0.1807	0.4893	0.79	0.6655	0.8146	1.9043	3.4111
0.30	0.1982	0.9165	0.1962	0.5226	0.80	0.6736	0.8000	1.9579	3.5051
0.31	0.2074	0.9250	0.2124	0.5569	0.81	0.6815	0.7846	2.0121	3.6020
0.32	0.2167	0.9330	0.2294	0.5921	0.82	0.6893	0.7684	2.0670	3.7021
0.33	0.2260	0.9404	0.2471	0.6284	0.83	0.6969	0.7513	2.1224	3.8062
0.34	0.2355	0.9474	0.2655	0.6657	0.84	0.7043	0.7332	2.1785	3.9144
0.35	0.2450	0.9539	0.2848	0.7040	0.85	0.7115	0.7142	2.2351	4.0276
0.36	0.2546	0.9600	0.3047	0.7433	0.86	0.7186	0.6940	2.2923	4.1466
0.37	0.2642	0.9656	0.3255	0.7836	0.87	0.7254	0.6726	2.3500	4.2722
0.38	0.2739	0.9708	0.3470	0.8249	0.88	0.7320	0.6499	2.4083	4.4057
0.39	0.2836	0.9755	0.3693	0.8672	0.89	0.7384	0.6258	2.4672	4.5486
0.40	0.2934	0.9798	0.3924	0.9104	0.90	0.7445	0.6000	2.5265	4.7033
0.41	0.3032	0.9837	0.4162	0.9546	0.91	0.7504	0.5724	2.5863	4.8724
0.42	0.3130	0.9871	0.4409	0.9997	0.92	0.7560	0.5426	2.6465	5.0602
0.43	0.3229	0.9902	0.4663	1.0459	0.93	0.7612	0.5103	2.7072	5.2727
0.44	0.3328	0.9928	0.4926	1.0929	0.94	0.7662	0.4750	2.7683	5.5182
0.45	0.3428	0.9950	0.5196	1.1410	0.95	0.7707	0.4359	2.8298	5.8119
0.46	0.3527	0.9968	0.5474	1.1900	0.96	0.7749	0.3919	2.8916	6.1785
0.47	0.3627	0.9982	0.5760	1.2400	0.97	0.7785	0.3412	2.9538	6.6695
0.48	0.3727	0.9992	0.6054	1.2908	0.98	0.7817	0.2800	3.0162	7.4063
0.49	0.3827	0.9998	0.6357	1.3427	0.99	0.7841	0.1990	3.0788	8.8261
0.50	0.3927	1.0000	0.6667	1.3956	1.00	0.7854	......	3.1416	......

The hydraulic jump in non-rectangular prismatic channels. The mathematical theory of the hydraulic jump in prismatic channels of other than rectangular cross section is based upon the fundamental law of conservation of linear momentum. In order that a stationary jump may form, the flow must first be at less than critical stage. The relation between the sequent depths cannot be expressed by a simple formula as for rectangular sections, but may be determined by a graphical method. Transpose the momentum equation to the following form:

$$wA_1\bar{y}_1 + \frac{wQV_1}{g} = wA_2\bar{y}_2 + \frac{wQV_2}{g} \qquad [402]$$

in which $wA_1\bar{y}_1$ and $wA_2\bar{y}_2$ represent the total hydrostatic pressure on cross sections before and after the jump, respectively. The other terms

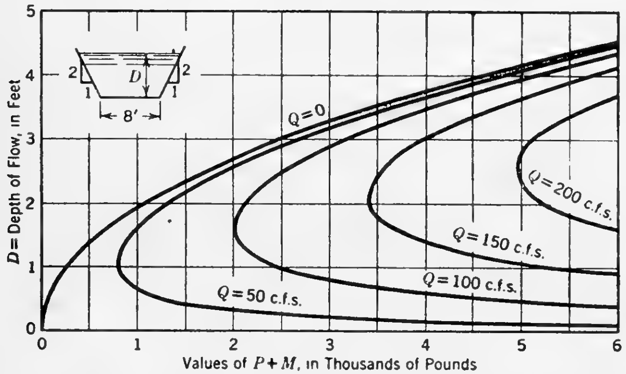


FIG. 403. $P + M$ as a Function of the Discharge and the Depth of Flow, in a Typical Trapezoidal Channel.

represent the momentum passing the section per second. This equation expresses the equality, before and after the jump, of the function $P + M$, given by

$$P + M = wA\bar{y} + \frac{wQV}{g} \qquad [403]$$

Since for a given discharge $P + M$ is a function of the depth only, curves representing values of the function for different constant values of Q may be plotted against the depth. An example is shown in Fig. 403. Depths before and after a jump are found at the two intersections of the curve for the given discharge with a constant value of $P + M$. The value of the depth at the point where the curve becomes parallel to the depth axis is the critical depth for that discharge. If a large number

of solutions is required, another diagram may be plotted in which the $P + M$ function has been eliminated in the same way that the total head was eliminated in preparing Fig. 402 from the data of Fig. 401. Several other methods are discussed by P. S. Hsing, who came to the conclusion that unless a very great number of solutions is to be made for the same channel, the amount of work involved in the preparation of charts or nomographs is not justified, and the problem should be solved by simple cut-and-try solution of equation (402).[1] Table 402 provides a solution for trapezoidal channels which is sufficiently accurate for most purposes.

In determining the location of a jump in a non-rectangular channel, the procedure described in Chapter III may be followed, but a slightly different method will usually be more convenient. Curves representing values of $P + M$ are plotted above the bottom of the channel, corresponding to the depths of flow of the upstream and downstream water surface profiles. The jump will form where the upstream and downstream values of $P + M$ are equal at a distance apart equal to the estimated length of the jump. This method differs from that of Chapter III in that two $P + M$ curves are plotted instead of one curve of sequent depths. Unless values of sequent depths are readily available it is more convenient.

Experimental work on the hydraulic jump in non-rectangular channels has been reported by Lane and Kindsvater, for circular channels, and by Posey and Hsing, for trapezoidal channels.[2] In both investigations, the observed relation of the sequent depths verified the conclusions of the momentum theory. The length of the jump increases rapidly as the deviation from the rectangular shape becomes marked. If the width after the jump is appreciably greater than the width before the jump, eddies will be formed at each side, modifying the character of the jump and reducing its capacity to dissipate most of the energy within a short space.

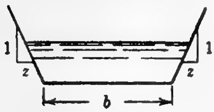
The depth after the jump in a circular conduit may be below the top of the section, or the conduit may flow full, under pressure if necessary, for the balance of pressure and momentum required by the theory. If the pipe does flow full, the air drawn into the jump is carried away downstream. As with the jump in a rectangular channel, the impact is violent. The velocity distribution becomes nearly uniform only a

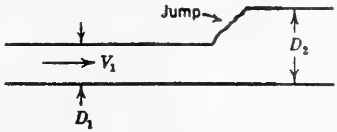
[1] "The Hydraulic Jump in Trapezoidal Channels," by Pei-su Hsing, thesis, State University of Iowa, 1937.

[2] "Hydraulic Jump in Enclosed Conduits," by E. W. Lane and C. E. Kindsvater, *Engineering News-Record*, Dec. 29, 1938; "Hydraulic Jump in Trapezoidal Channels," by C. J. Posey and P. S. Hsing, *Engineering News-Record*, Dec. 22, 1938.

TABLE 402

Sequent Depths in Trapezoidal Channels





$$H_1 = D_1 + \frac{V_1^2}{2g}$$

D_1/H_1	Values of D_2/H_1								
	$\frac{b}{zD_1} = 0$ Triangular sections	$\frac{b}{zD_1} = \frac{1}{3}$	$\frac{b}{zD_1} = 1$	$\frac{b}{zD_1} = 2$	$\frac{b}{zD_1} = 4$	$\frac{b}{zD_1} = 10$	$\frac{b}{zD_1} = 20$ $\frac{zD_1}{b} = 0.05$	$\frac{zD_1}{b} = 0.02$	Rectangular sections $\frac{zD_1}{b} = 0$
0.02	.132	.146	.158	.172	.192	.216	.242	.254	.268
0.04	.208	.228	.244	.260	.286	.320	.340	.358	.374
0.06	.270	.294	.312	.332	.360	.396	.416	.432	.448
0.08	.324	.350	.368	.396	.420	.460	.472	.492	.504
0.10	.370	.400	.418	.450	.476	.512	.528	.544	.552
0.12	.414	.444	.464	.496	.524	.558	.572	.584	.592
0.14	.454	.488	.508	.534	.564	.594	.608	.622	.628
0.16	.494	.526	.546	.572	.600	.630	.642	.652	.658
0.18	.530	.560	.584	.608	.634	.660	.670	.678	.682
0.20	.562	.594	.616	.638	.664	.686	.696	.702	.706
0.22	.592	.622	.646	.666	.690	.710	.716	.724	.726
0.24	.618	.652	.672	.692	.712	.730	.734	.740	.742
0.26	.646	.676	.696	.714	.732	.748	.750	.756	.758
0.28	.670	.700	.718	.734	.750	.764	.764	.768	.770
0.30	.694	.722	.738	.754	.766	.778	.778	.780	.780
0.32	.716	.740	.756	.772	.782	.788	.788	.788	.786
0.34	.736	.760	.774	.788	.794	.798	.796	.796	.792
0.36	.754	.776	.788	.800	.804	.806	.804	.800	.796
0.38	.772	.792	.802	.812	.814	.811	.806	.804	.799
0.40	.790	.806	.814	.820	.820	.814	.808	.805	.800
0.42	.804	.818	.826	.828	.825	.816	.810	.804	.799
0.44	.818	.830	.834	.834	.828	.816	.808	.802	.796
0.46	.830	.840	.842	.838	.830	.815	.806	.798	.792
0.48	.842	.848	.848	.842	.829	.812	.802	.794	.786
0.50	.852	.856	.852	.844	.828	.806	.796	.788	.780
0.52	.860	.862	.854	.844	.824	.800	.788	.780	.772
0.54	.866	.866	.854	.842	.818	.792	.780	.772	.762
0.56	.872	.866	.852	.838	.812	.784	.768	.760	.750
0.58	.876	.866	.850	.830	.802	.776	.756	.748	.738
0.60	.877	.864	.846	.822	.792	.760	.742	.734	.724
0.62	.878	.860	.840	.812	.780	.744	.728	.718	.708
0.64	.878	.854	.832	.800	.766	.728	.712	.702	.692
0.66	.876	.846	.822	.788	.750	.710	.694	.682	.674
0.68	.872	.838	.810	.772	.732	.692			
0.70	.866	.826	.796	.754	.712				
0.72	.858	.814	.778	.734					
0.74	.846	.798	.760						
0.76	.834	.778							
0.78	.818								
D_c/H	.800	.769	.750	.727	.706	.686	.677	.671	.667

few pipe diameters downstream from the jump. If enough air is not supplied above the jump, the pipe may fill with water and the jump cease to exist. The high velocity filaments will then continue downstream for a considerable distance. For an extremely high velocity, water vapor could presumably take the place of the air, with the jump forming as though air were being admitted. Limitations of the apparatus prevented testing this hypothesis. The condition would be undesirable in practice, because of the vibration and cavitation that would probably develop.

PROBLEMS

403. A trapezoidal channel with bottom width of 10 feet and side slopes of 4 vertical to 3 horizontal carries 500 c.f.s. flowing at a depth of 2 feet. If a jump forms in the channel, what will be the depth after the jump?

404. In experiments on the jump in a circular conduit 0.492 foot in diameter, the depth before the jump was 0.200 foot and the discharge was 0.667 c.f.s. The observed pressure head after the jump was 0.619 foot, measured above the center line of the pipe. Are these data in agreement with the momentum theory?

Effect of non-uniform velocity distribution. In the development of the relation between the alternate depths of flow at constant total head, and of the momentum theory giving the depths before and after a hydraulic jump, the velocity distribution has been assumed to be uniform throughout each section. This assumption gives results that are substantially in accordance with experimental results, if the distribution is like that found in uniform or gradually varied flow. If the distribution is markedly non-uniform, or if back currents or large eddies exist, the assumption may lead to conclusions considerably at variance with the facts. Such conditions are frequently found at sudden changes in cross section, particularly at enlargements.

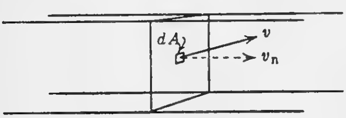


FIG. 404.

Figure 404 shows the velocity through a differential element dA of a cross section perpendicular to the axis of a flowing stream. In general, the velocity through dA would not be parallel to the axis of the stream. Let v represent the resultant velocity at dA and $v_n = v \cos \theta$ its component normal to the cross section, θ being measured from the normal pointing downstream. The average velocity across the entire section is represented by V, as before. The downstream flow through dA is equal to the component v_n times the area of the infinitesimal element, or

$$dQ = v_n dA$$

The change per second of the horizontal component of the momentum of the water passing through the area dA is

$$\frac{wv_n dA}{g} \cdot |v_n| = \frac{w}{g} v_n |v_n| dA$$

The absolute value sign is necessary to insure that those filaments for which the flow is upstream will be subtracted in getting the total for the entire cross section, which is

$$\frac{w}{g}\int v_n |v_n| dA$$

and not

$$\frac{wQV}{g}$$

as is usually assumed. Coriolis introduced a coefficient by which the latter value should be multiplied to equal the correct result.

$$\frac{w}{g}\int v_n |v_n| dA = \alpha \frac{wQ}{g} V \qquad \alpha = \frac{\int v_n |v_n| dA}{V^2 A} \qquad [404]$$

For a non-uniform distribution, α will always be greater than unity.

The kinetic energy carried through dA, per unit of time, will be

$$\frac{1w}{2g} \cdot v_n dA \cdot v^2$$

which expression should be integrated over the whole cross section to determine the total kinetic energy of the water passing downstream through the section per unit of time. Since this is customarily written

$$\frac{wQ}{2g} V^2$$

a coefficient is again needed, defined by

$$\frac{w}{2g}\int v_n v^2 dA = \alpha' \frac{wQ}{2g} \cdot V^2 \qquad \alpha' = \frac{\int v_n v^2 dA}{V^3 A} \qquad [405]$$

Some writers advocate use of this coefficient in the application of Bernoulli's theorem

$$Z_1 + \frac{p_1}{w} + \alpha_1' \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{w} + \alpha_2' \frac{V_2^2}{2g} + h_f \qquad [406]$$

For normal velocity distributions the value of α' is nearly unity, so that to introduce this coefficient has very little effect. If one of the

values of α' is high, indicating a markedly non-uniform distribution, the introduction of this coefficient may cause erroneous results. Consider a stream flowing at less than the critical velocity, with partly closed gate obstructing the flow as shown in Fig. 405. At a section

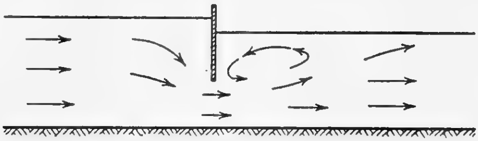


FIG. 405. Water Flowing Under a Gate.

immediately downstream from the gate, the horizontal component of the velocity is zero over part of the cross section, say half. Assuming that the velocity is uniform over the other half of the section, the values of α and α' are

$$\alpha = \frac{(2V)^2 \cdot \frac{1}{2}A}{V^2 A} = 2 \qquad \alpha' = \frac{(2V)^3 \cdot \frac{1}{2}A}{V^3 A} = 4$$

with V representing the average velocity over the entire cross section. It would lead to incorrect results, however, to use this value of α' in Bernoulli's theorem at an *upstream* section, assuming that the velocity head so obtained could be reconverted into pressure or elevation head. The law of conservation of energy still holds, but in this case a large part of the kinetic energy is not recoverable. It will be dissipated by the eddies which are generated at the plane of contact of the high velocity sheet and the stagnant water above, and are carried downstream with the current. On the other hand, the use of a high value of α' for the *downstream* section in Bernoulli's equation is permissible, if warranted by actual conditions. The coefficient α', however, is chiefly useful in laboratory investigations where the energy loss between the two sections is the unknown to be determined.

EXERCISE

Prove that for a non-uniform distribution of velocity, α is greater than unity.

Channel in which the flow is critical at any stage. For such a channel the velocity head must equal half the average depth at all stages, or

$$h_v = \frac{A}{2T}$$

If we assume the total head to remain constant, the level of the water surface may be determined by its distance below the elevation rep-

resenting the total head. This distance is $h_v = V^2/2g$. If the width of water surface, T, can be expressed as a function of h_v the shape of the channel will be determined. Substituting $A = Q/V$ in the above expression

$$h_v = \frac{Q}{2TV}$$

or

$$T = \frac{Q}{2\sqrt{2}g^{1/2}h_v^{3/2}}, \quad h_v = \frac{Q^{2/3}}{2g^{1/3}T^{2/3}}, \quad \text{since} \quad V = \sqrt{2gh_v}$$

Figure 406 shows the shape of this channel. The theoretical curves come together an infinite distance downward, but since the area enclosed is not infinite, it is possible to construct a rectangular bottom, having the correct area and surface width. The stage must not be allowed to descend into this bottom. There is only one such channel

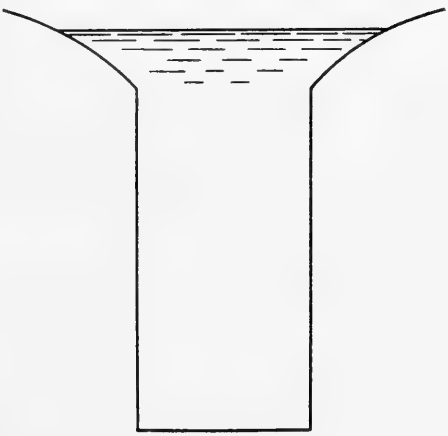


FIG. 406.

For all depths of flow above the rectangular portion of this channel, the flow should be critical.

for a given discharge. As far as is known to the writers, none has ever been built. It would be interesting to build one, for except for the influence of lateral components of velocity, the depth could change over wide limits without energy being supplied or taken away. The surface should be very unstable.

Channel in which the hydraulic radius remains constant. This channel has been an object of interest because of the theoretical possibility of constructing an earth canal that would be non-scouring and non-silting over a wide range of stages. Representing the hydraulic

radius by R and the area and wetted perimeter by A and P,

$$R = \frac{A}{P} = \frac{A + dA}{P + dP} = \frac{dA}{dP} = \frac{xdy}{\sqrt{dx^2 + dy^2}}$$

The general solution of this differential equation is

$$y = R \cosh^{-1} \frac{x}{R} + C$$

Axes may be chosen so as to eliminate the constant, so that

$$y = R \cosh^{-1} \frac{x}{R} \qquad x = R \cosh \frac{y}{R}$$

Inasmuch as the curve is not closed at the bottom, even at infinity, the " channel " will not hold water, and we may conclude that a channel for which the hydraulic radius is a constant for *any* depth does not

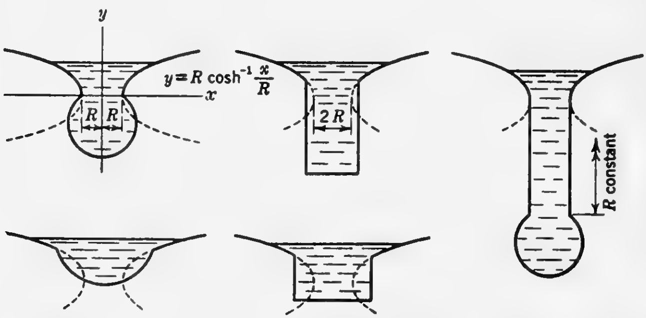


FIG. 407. Channels in which the Hydraulic Radius is Constant for all Depths of Flow Above the Bottom Sections of Elementary Shape.

exist. If we allow the restriction that the depth of flow be always above a certain level, a bottom may be fitted to the curve. It is necessary for the bottom to have a hydraulic radius equal to R, but it may join the upper curve at any horizontal line. Figure 407 shows circular and rectangular elements of the correct dimensions fitted to the curve to form channels which have constant hydraulic radius as long as the water level is above the top of the elementary section.

This, however, is not the only possible solution. Returning to the differential equation

$$\frac{dy}{dx} = \frac{R}{\sqrt{x^2 - R^2}} = p$$

where p represents dy/dx. Clearing the equation of fractions and radicals

$$p^2(x^2 - R^2) - R^2 = 0$$

Differentiating with respect to p,

$$p(x^2 - R^2) = 0$$

Now eliminating p between these two equations we obtain the singular solution of the differential equation,

$$x = \pm R$$

which is the equation of the pair of tangents common to the family of curves $y = R \cosh^{-1} x/R + C$ for different values of C. This solution satisfies the differential equation as well as the physical requirements of the problem. It, too, requires a bottom section. The two solutions may be used in combination, as shown in Fig. 407.

The channel of constant hydraulic radius is clearly impractical for earthen canals, for the only shapes in which the bottom portion would be stable allow but little variation of the water surface. It was once thought to be practicable, owing to the error of an early writer in fitting a semicircular bottom to the catenary curve at the point where the sides are vertical. The channel does, however, offer an experimental approach for the determination of the adequacy of the hydraulic radius as the sole shape parameter in the friction velocity formulas.

CHAPTER V

THE MOVING HYDRAULIC JUMP

The moving hydraulic jump is a transitory phenomenon closely related to the stationary hydraulic jump. The bore, which occurs in some tidal rivers and estuaries, is a moving hydraulic jump. The surge is a special case of the moving hydraulic jump. The action of the ocean surf on smooth beaches provides a ceaseless natural example of the moving hydraulic jump. As the height of a jump moving in still water approaches zero, the water surface does not break, and the jump becomes indistinguishable from the small "solitary positive wave" observed by J. Scott Russell.

By experimental means, Russell determined the correct formula for the velocity of the wave in still water.[1] Later Bazin determined its velocity in moving water. Bazin also experimented with surges and moving jumps of appreciable height.[2]

The equations for the moving hydraulic jump may be derived from those for the stationary jump by careful application of the principle of relative velocities, or they may be derived by direct application of the fundamental laws of continuity and change of momentum. Figure 501 (a) represents a vertical longitudinal section through a hydraulic jump moving to the right with velocity V_0. The channel is rectangular, with horizontal bottom, and friction is to be neglected. It is convenient to consider a unit width. The velocity distribution, at sections before and after the jump, is assumed to be uniform, and the positive direction for all velocities is toward the right. If D_1, D_2, V_1, and V_2 are constant, then V_0 will also be constant.

In Fig. 501 (a) at a particular instant the jump is supposed to be at *cd*. Consider the mass of water *acdfgja*, including the jump and contained between the vertical cross sections *aj* and *fg*. This is shown as the shaded area in Fig. 501 (b). After a brief interval of time dt all this water will have moved towards the right, to the position *bkemnhb*, shown by the shaded area in Fig. 501 (c). The jump will have moved from *cd* to *ke*, a distance equal to $V_0 dt$; the cross section *aj* to the

[1] Report of the fourteenth meeting of the British Association for the advancement of science. London, 1845.

[2] "Recherches Hydrauliques," Darcy and Bazin, Paris, 1865.

position *bh*, a distance equal to V_1dt; and the cross section *fg* to *mn*, a distance equal to V_2dt.

The law of continuity may be applied in either of two ways. Considering the two cross sections *bh* and *fg*, Fig. 501 (*a*), there is less water contained between these two cross sections at the end of the interval *dt*, when the jump is at *ke*, than there was at the beginning of the

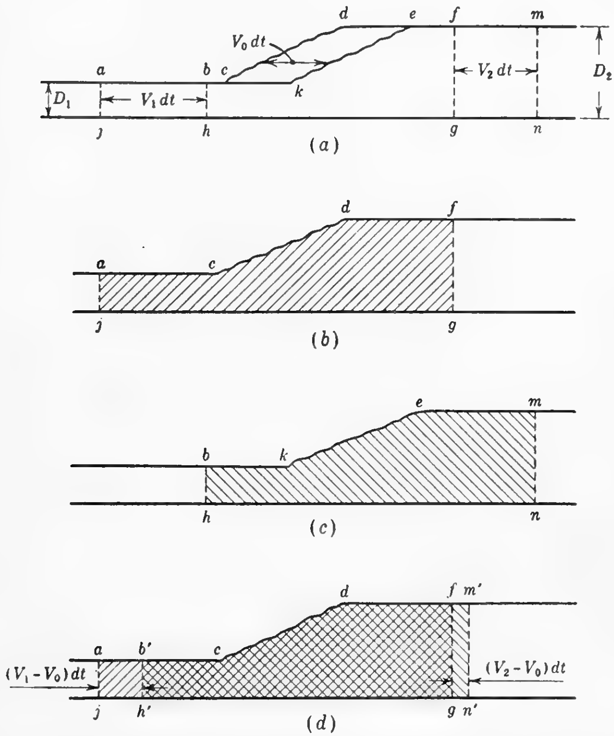


FIG. 501. Vertical Longitudinal Section through a Moving Hydraulic Jump. (All velocities are toward the right.)

interval, when the jump was at *cd*. Therefore, the amount of water flowing out across the section *fg*, equal to D_2V_2dt, is greater than the amount flowing in across the section *bh*, equal to D_1V_1dt; and greater by an amount exactly equal to the reduction in storage between cross section *bh* and *fg*. The reduction in storage is represented by the area *cdek*, whose height is $D_2 - D_1$ and whose length is V_0dt. Therefore,

$$D_2V_2 = D_1V_1 + (D_2 - D_1)V_0 \qquad [501]$$

The other way to apply the law of continuity is as follows: Move the shaded area of Fig. 501(c) to the left an amount equal to $V_0 dt$, the distance that the jump has traveled, and then superpose this area upon the area shown in Fig. 501(b) so that ek will coincide with dc, as shown in Fig. 501(d). Then bh will likewise have moved to the left a distance $V_0 dt$ to $b'h'$, and mn to $m'n'$. The two shaded areas as shown in Fig. 501(d) will only partially coincide, leaving at the left the area $ab'h'j$ and at the right $fm'n'g$, which by the law of continuity must be equal. Therefore,

$$D_1(V_1 - V_0) = D_2(V_2 - V_0) \qquad [502]$$

which is the equivalent of equation (501) by a simple algebraic transposition.

To apply the law of change of momentum, the difference between the momentum of the mass of water shown in Fig. 501(b) and the momentum of the same mass of water in its later position Fig. 501(c) must be found. The momentum of a mass is found by taking the sum of the momenta of its parts. When the two masses are superposed as in Fig. 501(d) the portion between $b'h'$ and fg is common to both masses. Therefore, the desired change in momentum is the momentum of the mass $ab'h'j$ minus the momentum of the equal mass $fm'n'g$, or it is the mass $ab'h'j$ multiplied by the change in velocity $V_1 - V_2$.

The momentum of any body can be changed only as the result of an external unbalanced force acting upon the body. The momentum of the mass of water shown in Fig. 501(c) is less than the momentum of the same mass in the position Fig. 501(b), because the hydrostatic force acting towards the left against the face fg is greater than the hydrostatic pressure acting towards the right against the face aj. The difference between these two pressures is the unbalanced force which reduces the momentum of the mass of moving water while it travels from position Fig. 501(b) to position Fig. 501(c). The pressure against the face fg is $\frac{w}{2} D_2^2$, against the face aj, $\frac{w}{2} D_1^2$.

The law of the change of momentum is that the unbalanced force is equal to the time rate of change of momentum. In this case the weight of water to be considered is the weight of $ab'h'j$, equal to $w\,D_1(V_1 - V_0)dt$, the change in velocity is $V_1 - V_2$, and the time during which the change takes place is dt.

Therefore,

$$\frac{w}{2}(D_2^2 - D_1^2) = \frac{wD_1(V_1 - V_0)dt(V_1 - V_2)}{gdt}$$

or as it may be written by the aid of equation (502)

$$\tfrac{1}{2}g(D_2^2 - D_1^2) = D_1(V_1 - V_0)V_1 - D_2(V_2 - V_0)V_2 = D_1(V_1 - V_0)(V_1 - V_2) = D_2(V_2 - V_0)(V_1 - V_2) \qquad [503]$$

The pair of simultaneous equations (502) and (503) represent the two laws, namely, the law of continuity and the law of change of momentum and contain the whole theory for the moving hydraulic jump for the case being considered. These equations contain five variables, any three of which may be considered the independent variables. If values for three variables are given or known, the values of the other two will be fixed.

From equations (502) and (503) any one of the five variables may be eliminated, giving five different equations each containing four variables. Each of these equations may be solved for any one of the variables, giving a total of twenty different forms in which these equations might conceivably be written. To write all these forms requires in some cases the solution of a quadratic, and in others the solution of a cubic equation. Furthermore, various other variables may be introduced into these equations, such as the velocity heads $V_1^2/2g$, $V_2^2/2g$, the total heads, $D_1 + (V_1^2/2g)$, $D_2 + (V_2^2/2g)$, the height of the jump $D_2 - D_1$, the difference between the total heads, or the loss of energy in the jump, as well as ratios of these variables. Evidently a great number of different equations can be thus formed, if desired or needed for any purpose, by ordinary algebraic transformations.

Graphical solution of hydraulic jump equations. It is a difficult matter, in general, to construct a diagram from which functions of three independent variables may be conveniently read. In this case by taking advantage of the symmetrical nature of equations (502) and (503) it is possible to construct such a diagram in a form convenient for general use, as shown in Fig. 502. The process is as follows: Eliminating D_2 and solving for $V_2 - V_0$ gives

$$V_2 - V_0 = \frac{\frac{1}{4}gD_1}{V_1 - V_0} + \sqrt{\tfrac{1}{2}gD_1 + \left(\frac{\frac{1}{4}gD_1}{V_1 - V_0}\right)^2} \qquad [504]$$

Taking first when $V_0 = 0$, we have

$$V_2 = \frac{gD_1}{4V_1} + \sqrt{\tfrac{1}{2}gD_1 + \left(\frac{gD_1}{4V_1}\right)^2} \qquad [505]$$

Similarly, eliminating D_1 and solving for $V_2 - V_0$ gives

$$V_2 - V_0 = \frac{\frac{1}{2}gD_2(V_1 - V_0)}{(V_1 - V_0)^2 - \frac{1}{2}gD_2} \qquad [506]$$

which, when $V_0 = 0$, reduces to

$$V_2 = \frac{\frac{1}{2}gD_2V_1}{V_1^2 - \frac{1}{2}gD_2} \qquad [507]$$

From equations (505) and (507) numerical values were calculated by means of which the curved lines on the right-hand part of Fig. 502 were plotted. Values of V_1 and V_2 are read along the coordinate axes, and the two sets of curved lines represent equal values of D_1 and D_2 respectively. The axes for V_1 and V_2 are inclined at 120 degrees, instead of being rectangular, for two reasons: first, in order to utilize the space more efficiently; second, in order to obtain subsequently the same scale for V_0 as is used for V_1 and V_2. It should be carefully noted that lines for constant values of V_1 run from the bottom of the diagram upwards and towards the left, while lines for constant values of V_2 run from the bottom of the diagram upwards and towards the right.

The diagram as so far described suffices for the solution of problems on the stationary jump. Of the four variables V_1, V_2, D_1, and D_2, values for any two may be given. These values will locate a point on the diagram, where the corresponding values of the other two variables may be instantly read. For example, assume that the given data are $V_1 = 15$ feet per second and $D_1 = 2$ feet. These locate the point marked P on Fig. 502, showing that $V_2 = 6.8$ feet per second and $D_2 = 4.4$ feet. It is apparent that the given data might have been $V_1 = 15$ and $V_2 = 6.8$, locating the same point, from which the corresponding values of D_1 and D_2 could be read.

To use the diagram for the solution of problems on the moving hydraulic jump it is necessary to introduce the fifth variable V_0. This is done by moving the point on the diagram horizontally a distance equal to V_0 measured on the velocity scale along the bottom of the diagram. For example, suppose the given data to be $D_1 = 2$, $D_2 = 4.4$, $V_0 = 5$. The values of D_1 and D_2 give the point P in the diagram. From P a distance is measured horizontally to the right equal to 5 on the velocity scale, locating the point Q, from which may be read the values $V_1 = 20$ feet per second, $V_2 = 11.8$ feet per second. It is evident that the operation would be equally easy if the given values were V_1, V_2, and V_0. It is not quite so convenient, however, when such data are given as $D_1 = 2$, $V_1 = 20$, and $V_0 = 5$. In such a case it is necessary by careful inspection or by using a movable horizontal scale to determine just where the line for $D_1 = 2$ and the line for $V_1 = 20$ are apart, measured horizontally, a distance exactly equal to the given value of V_0. This can be done by the exercise of some care and at the cost of a little inconvenience. If the given value of V_0 is negative, then

of course the distance for V_0 must be measured in the opposite direction, that is, horizontally from the intersection of the D_1 and D_2 lines towards the left to the intersection of the V_1 and V_2 lines. This may lead to negative values of V_1 and V_2, which, of course, are quite possible.

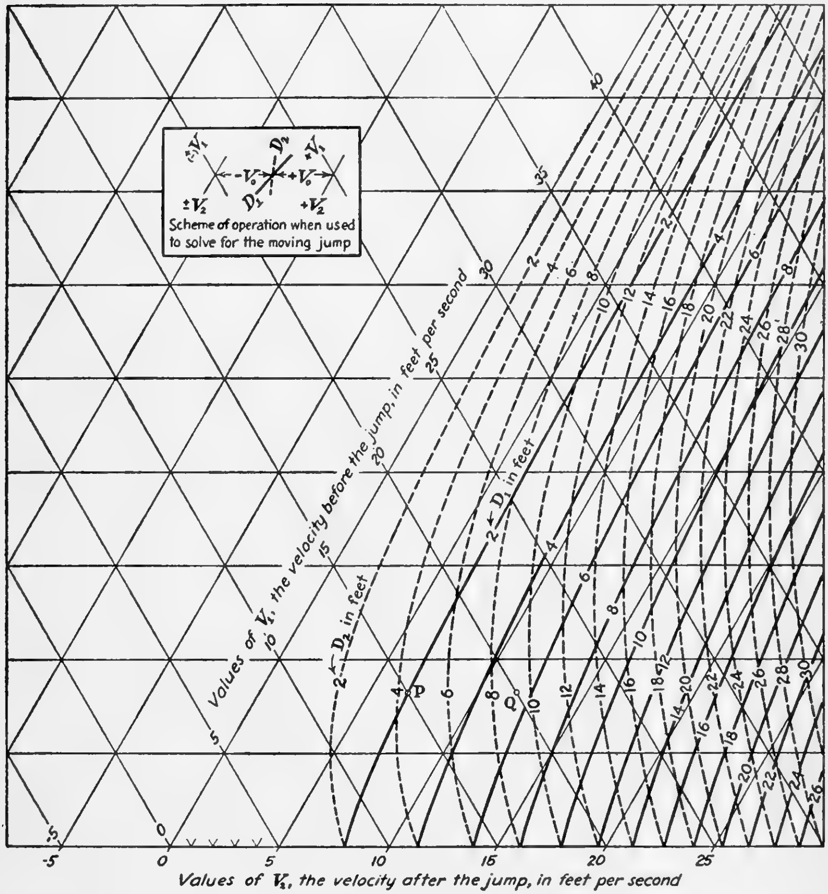


FIG. 502. Diagram for the Hydraulic Jump in a Rectangular Channel.

May be used for the moving jump. Scale for V_0 same as for V_1 and V_2.

Various cases of the moving jump. In the kinds of moving jump most frequently occurring in nature, such as the ocean surf and the tidal bore, V_0 is negative. It is found rather difficult in the laboratory to produce a satisfactory moving jump having much velocity in the positive direction. Apparently this is because the high velocities necessary produce such high friction as to interfere with the formation of the

typical jump. Many of the rules and limitations applicable to the stationary jump are no longer true for the moving jump, which may go through such a range of variations that it is difficult to describe them all.

In considering the different cases only positive values of D_1 and D_2 will be considered for there seems to be no useful significance to negative values. Furthermore, to produce the jump a clash of currents seems to be necessary; hence if D_1 is assumed to be to the left of the line of separation of the two currents and D_2 to the right, as shown in Fig. 501, then V_1 must be algebraically greater than V_2 or there could be no clash. It follows then that $V_1 - V_0$ must always be algebraically greater than $V_2 - V_0$, and from equation (502), D_2 must be greater than D_1. Equations (502) and (503) show that D_1 and D_2 can be interchanged if V_1 and V_2 are interchanged at the same time. However, as explained above, there would then be no clash unless directions were also reversed, and doing this leads to no additional information. To economize space, therefore, Fig. 502 is drawn to include only the area where D_1 is less than D_2; a symmetrical lower half is omitted.

When V_1, V_2, and V_0 are all positive, V_0 cannot be greater than V_2. If V_0 is negative there are five possible cases that may be considered. First, V_1 and V_2 may both be positive. Second, V_2 may be zero while V_1 is still positive. This is the case when a gate is suddenly closed in a canal containing flowing water. Third, V_2 may become negative while V_1 is still positive. This is the case in the surf and the tidal bore. Fourth, with V_2 negative V_1 may become zero. Fifth, both V_1 and V_2 may be negative. This may occur in the case of a flood wave traveling down a stream. The special case in which the height of the jump approaches zero is the "solitary wave" observed by J. Scott Russell.

PROBLEMS

501. Fill in the blank spaces in the following table in which each horizontal line represents a moving hydraulic jump in a frictionless rectangular channel with horizontal botton.

D_1	V_1 ft./sec.	D_2 ft.	V_2 ft./sec.	V_0 ft./sec.
8		20		+5
	+25		+20	+5
	+10	5	− 5	
8	+15			−5

502. A gate is to be suddenly dropped into place closing a rectangular channel 6 feet deep and 10 feet wide, in which 200 c.f.s. are flowing at a depth of 4 feet. Will the canal overflow? Assuming the sides of the channel to be sufficiently high to prevent it from overflowing, what will be the velocity of the traveling jump produced?

CHAPTER VI

BACKWATER CURVES — INTRODUCTORY

To the layman, " backwater " means the water retarded above a dam or backed up into a tributary by a flood in the main stream. He has probably never thought of the existence of a transition between the level pool and the unretarded stream above. Frequently, this transition takes the shape of a long, smooth curve which is continuously concave upward. It is this " backwater curve " that first commanded the attention of hydraulicians. When it was found that other types of surface profiles existed, an attempt was made to supply each with a suitable name. Ultimately, however, twelve different types of surface curves were found to be possible, and the selection of a distinctive and appropriate name for each became well-nigh impossible. The term backwater curve has come to be applied to all of them. The various types are distinguished by criteria which will be explained in this chapter.

Longitudinal water-surface curves, in general, include not only backwater curves, but also profiles through the hydraulic jump, over weirs, around bends, and past changes in the cross section of the stream. In phenomena due to sudden changes in direction or cross section, vertical or lateral components of velocity play such a large part that they cannot be ignored. On the other hand, the profile of a backwater curve is always comparatively flat, changes in section are gradual, and vertical and lateral components of velocity may be neglected without introducing appreciable error.

We shall consider only steady flow, that is, the condition in which the flow at each cross section remains constant with respect to time. For the existence of a backwater curve the flow is non-uniform. The study of backwater curves is complicated by the large number of independent variables. It seems desirable, therefore, to study first the simplest cases having practical utility.

Backwater curves in a frictionless rectangular channel. It was shown in Chapter II that if a frictionless channel has a level bottom, in general, there can be no variation in depth of flow or in velocity. The backwater curve reduces to a simple, horizontal, straight line. If the bottom slopes, surface curves are possible, in accordance with Bernoulli's

theorem. Before proceeding with the analysis of these curves it is necessary to list the notation to be used.

y = depth of water flowing in the channel at any section.

x = horizontal distance along the channel from some arbitrary origin to the section where the depth is y.

V = average velocity of the flowing water, considered positive when toward the right.

Q = volume of water flowing per foot width of channel. Q is constant, for the flow is steady.

y_c = depth of critical flow. For a rectangular channel $gy_c^3 = Q^2$. (See Chapter II.)

S_w = slope of the water surface. The slope is customarily considered to be positive for a downward slope in the direction of flow. This is illustrated by the Chezy formula, in which the slope must be positive. Though mathematically awkward, this sign convention has roots so deep in hydraulic practice that it seems best to continue its use, even where to do so introduces difficulties.

S_0 = slope of the channel bottom, considered positive if downward in the direction of flow.

$\frac{dy}{dx}$ = slope of the water surface relative to the bottom. In order to conform with the usual convention of the calculus, this is taken as positive if the depth increases in the direction of flow.

For flow without friction to satisfy Bernoulli's theorem, the slope of the water surface must equal the rate of change of velocity head with respect to distance along the stream, or

$$S_w = \frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{V\,dV}{g\,dx}$$

The flow is steady, which means that Q is constant, so that the law of continuity must also be satisfied, and

$$V = \frac{Q}{y} \qquad dV = \frac{-Q\,dy}{y^2}$$

Substituting these relations in the above equation,

$$S_w = \frac{-Q^2\,dy}{y^3 g\,dx}$$

But

$$\frac{Q^2}{g} = y_c^{\,3}$$

so that

$$S_w = \frac{d}{dx}\left(\frac{V^2}{2g}\right) = -\left(\frac{y_c}{y}\right)^3 \frac{dy}{dx} \qquad [601]$$

By geometry,

$$S_w = S_0 - \frac{dy}{dx} \qquad [602]$$

so that

$$S_0 = \frac{dy}{dx} - \left(\frac{y_c}{y}\right)^3 \frac{dy}{dx}$$

$$S_0\, dx = \left[1 - \left(\frac{y_c}{y}\right)^3\right] dy$$

Integrating, and letting $x = 0$ when $y = y_c$

$$S_0 x = y + \frac{y_c^3}{2y^2} - \frac{3}{2} y_c \qquad [603]$$

The graph of this equation is shown in Fig. 601. It is evident that the two branches of the curve shown are different backwater curves, that is, a water-surface profile could not possibly follow continuously through

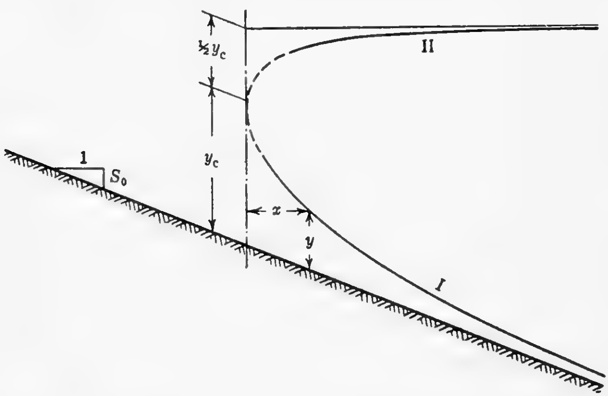


FIG. 601. Backwater Curves in a Frictionless Rectangular Channel.

both. Note that one branch is asymptotic to the bottom, while the other is asymptotic to a horizontal line at an elevation $y_c/2$ above the critical depth. This corresponds to a complete regain of the velocity head. Both curves cross the critical depth vertically. Except in the range near the critical depth, these curves would provide a close approximation to the surface curves in a steep rectangular channel, over dis-

tances sufficiently short for the effect of friction to be negligible. The approximation would be poor near the critical depth in any case, for in deriving the equation the effect of vertical components of velocity was neglected. These become important when the value of dy/dx is large, as it is near the critical depth.

Another branch of the curve representing the mathematical equation lies below the bottom of the channel. It is omitted because it has no real meaning in this connection.

PROBLEMS

601. Prove, from the equation of the curve of Fig. 601, that branch II has a horizontal asymptote at the location shown on the graph.

602. What determines whether water flowing at the critical depth will follow branch I or II?

Bresse's backwater curves. Theoretical backwater curves including the effect of friction as well as that of the change of kinetic energy were first treated by Bresse, who considered only an infinitely wide rectangular channel, and used the Chezy formula for the evaluation of the effect of friction.[1] The Chezy formula is now outmoded, and it is known that the shape of the channel may have an appreciable effect, so that Bresse's curves cannot be used if the greatest accuracy is desired. However, they are by far the most convenient for computation, and find use when the available data or the results desired do not justify a more accurate procedure. Where the channel is wide and flat, and the constant in Chezy's formula does not vary over too wide a range, the results from Bresse's method are as close to the actual observed values as are results from the best of the other methods. The simplicity of Bresse's method makes it useful in studying the various cases of the backwater curves. Curves plotted from the tables prepared by Bresse are similar in general shape to those plotted from tables of the more complicated backwater functions.

Bresse's method makes use of Chezy's formula, which is intended to apply only to steady uniform flow, when the water surface is parallel to the bottom. In that case the slope of the bottom, the slope of the water surface, and the slope representing the rate at which head is being used up in overcoming friction are all the same. If the flow is non-uniform, all three slopes may be different. At a given section of a stream in which the flow is non-uniform, we assume that the *friction* slope is given correctly by Chezy's formula, so that if the channel is assumed to

[1] J. A. C. Bresse, "Méchanique Appliquée," v. 2, Mallet-Bachelier, Paris, 1860.

be rectangular, and infinitely wide,

$$S_f = \frac{V^2}{C^2 y} = \frac{Q^2}{C^2 y^3} \qquad [604]$$

The subscript f distinguishes the friction slope from the water surface slope S_w and the bottom slope S_0. To use Chezy's formula in this way is to assume that the rate of loss of energy through friction, at a given section, is the same in non-uniform flow as it would be for uniform flow at the depth of flow existing at that section. This is not strictly true because expanding flow tends to be more turbulent, and contracting flow less turbulent, than uniform flow. The error from this source is small when the rate of contraction or expansion is small, which is the case with backwater curves in uniform channels.

For a given quantity of flow, channel shape, slope, and roughness, a certain depth of flow will be just right to maintain uniform flow. As noted before, the surface slope, the friction slope, and the bottom slope are equal for uniform flow, so that for an infinitely wide rectangular channel this depth, termed the normal (or neutral) depth, is determined by Chezy's formula to be

$$y_n = \frac{V^2}{C^2 S_0} = \sqrt[3]{\frac{Q^2}{C^2 S_0}} \qquad [605]$$

The normal depth is of great importance. Together with the critical depth and the bottom slope, it forms the basis for the classification of all possible backwater curves, according to the following outline:

- Channel bottom sloping downward.
  - y_n greater than y_c, mild slope.
    - *M1*, y greater than y_n.
    - *M2*, y less than y_n but greater than y_c.
    - *M3*, y less than y_c.
  - y_n less than y_c, steep slope.
    - *S1*, y greater than y_c.
    - *S2*, y less than y_c but greater than y_n.
    - *S3*, y less than y_n.
  - y_n equal to y_c, critical slope.
    - *C1*, y greater than y_c.
    - *C3*, y less than y_c.
- Channel bottom horizontal.
    - *H2*, y greater than y_c.
    - *H3*, y less than y_c.

Channel bottom sloping upward, adverse slope.

A2, y greater than y_c.

A3, y less than y_c.

Boundary cases, such as that between the *M1* and *M2* curves, are not included, for they represent either uniform flow, or conditions impossible for continued flow. The twelve cases are shown in Fig. 602.

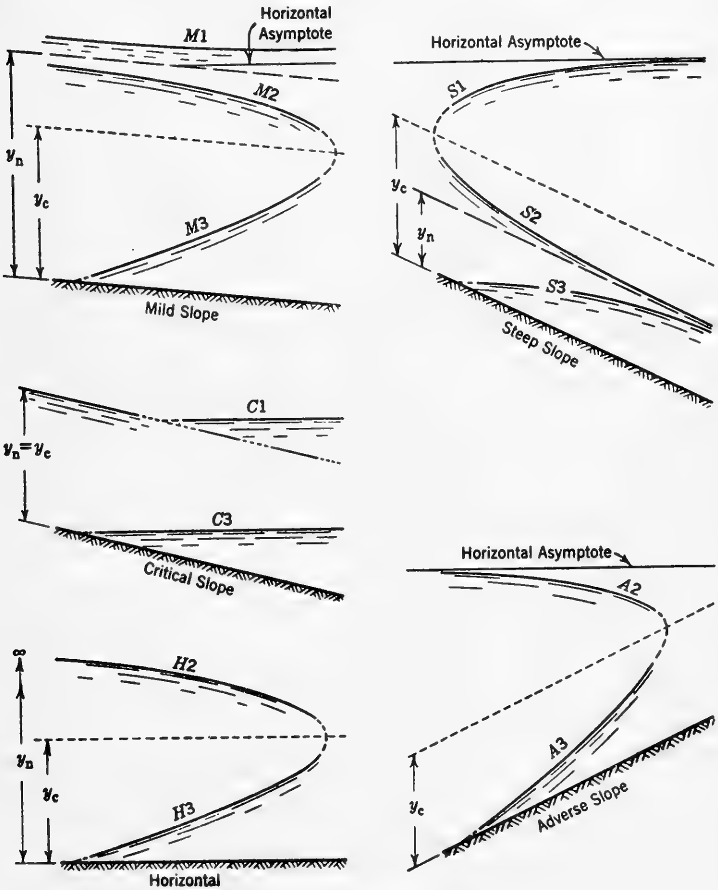


FIG. 602. Backwater Curves.

Flow is from left to right in every case.

For the flow to satisfy Bernoulli's theorem, the slope of the water surface must equal the rate of change of velocity head with respect to distance along the stream, plus the friction slope, so that

$$S_w = \frac{d}{dx}\left(\frac{V^2}{2g}\right) + S_f$$

By geometry $S_w = S_0 - dy/dx$. Substituting this value of S_w in the above equation

$$S_0 - \frac{dy}{dx} = \frac{d}{dx}\left(\frac{V^2}{2g}\right) + S_f \qquad [606]$$

Equation (606) is the general differential equation for all of the backwater curves shown in Fig. 602. Substituting the value of S_f found by eliminating Q^2/C^2 between equations (604) and (605) and the value of $\frac{d}{dx}\left(\frac{V^2}{2g}\right)$ from equation (601), we obtain

$$S_0 - \frac{dy}{dx} = -\left(\frac{y_c}{y}\right)^3 \frac{dy}{dx} + \left(\frac{y_n}{y}\right)^3 S_0 \qquad [607]$$

which is the differential equation for the backwater curves in an infinitely wide rectangular channel, friction evaluated by Chezy's formula. It has the integral solution

$$x = \frac{y_n z}{S_0} - y_n\left(\frac{1}{S_0} - \frac{C^2}{g}\right)\left[\frac{1}{6}\log_e \frac{z^2 + z + 1}{(z-1)^2} - \frac{1}{\sqrt{3}}\operatorname{cotan}^{-1}\frac{2z+1}{\sqrt{3}}\right] \qquad [608]$$

in which z stands for y/y_n. Tables of the function contained within the brackets were computed by Bresse for positive values of z. Table 601 gives values of this function for positive and negative values of z.[2]

Cases of the backwater curves. All possible kinds of backwater curves are shown in Fig. 602. The curves are plotted to an exaggerated vertical scale, and have been shown dotted near the critical depth as a reminder that this portion of the curve does not possess the same degree of accuracy as the rest of the curve, owing to neglect of the vertical components of velocity. In several cases the curve has either a beginning or an abrupt ending at the critical depth. In plotting the curves by the use of Table 601, *M2* and *M3* will appear as one continuous curve, and similarly with *S1* and *S2*. This is because mathematically they are represented by the same continuous function. In steady flow in a uniform channel at constant grade it would be impossible for any backwater curve to cross the critical depth. The water surface could cross the critical depth only through a hydraulic jump.

Many of the peculiar features of the curves shown in Fig. 602 may

[2] In evaluating Part 2 of the table, a constant, $\pi/3\sqrt{3}$, was added to $\Phi[z]$ to prevent the function from changing sign. Similarly, the constant $\pi/\sqrt{3}$ was added in computing the values in Part 3. Since no backwater curve utilizes more than one part of the table, the introduction of a constant in this manner does not affect the use of the table, except to make errors less likely.

TABLE 601

Bresse's Backwater Function

Part 1

For *M1*, *S1*, and *S2* curves

z	$\Phi[z]$	z	$\Phi[z]$	z	$\Phi[z]$	z	$\Phi[z]$
1.000	∞	1.054	0.8714	1.29	0.3816	2.30	0.0978
1.001	2.1837	1.056	0.8599	1.30	0.3731	2.35	0.0935
1.002	1.9530	1.058	0.8489	1.31	0.3649	2.40	0.0894
1.003	1.8182	1.060	0.8382	1.32	0.3570	2.45	0.0857
1.004	1.7226	1.062	0.8279	1.33	0.3495	2.50	0.0821
1.005	1.6486	1.064	0.8180	1.34	0.3422	2.55	0.0788
1.006	1.5881	1.066	0.8084	1.35	0.3352	2.60	0.0757
1.007	1.5371	1.068	0.7990	1.36	0.3285	2.65	0.0728
1.008	1.4929	1.070	0.7900	1.37	0.3220	2.70	0.0700
1.009	1.4540	1.072	0.7813	1.38	0.3158	2.75	0.0674
1.010	1.4192	1.074	0.7728	1.39	0.3098	2.80	0.0650
1.011	1.3878	1.076	0.7645	1.40	0.3039	2.85	0.0626
1.012	1.3591	1.078	0.7565	1.41	0.2983	2.90	0.0604
1.013	1.3327	1.080	0.7487	1.42	0.2928	2.95	0.0584
1.014	1.3083	1.082	0.7411	1.43	0.2875	3.00	0.0564
1.015	1.2857	1.084	0.7337	1.44	0.2824	3.1	0.0527
1.016	1.2645	1.086	0.7265	1.45	0.2775	3.2	0.0494
1.017	1.2446	1.088	0.7194	1.46	0.2727	3.3	0.0464
1.018	1.2259	1.090	0.7126	1.47	0.2680	3.4	0.0437
1.019	1.2082	1.092	0.7059	1.48	0.2635	3.5	0.0412
1.020	1.1914	1.094	0.6993	1.49	0.2591	3.6	0.0389
1.021	1.1755	1.096	0.6929	1.50	0.2548	3.7	0.0368
1.022	1.1603	1.098	0.6867	1.52	0.2466	3.8	0.0349
1.023	1.1458	1.100	0.6806	1.54	0.2389	3.9	0.0331
1.024	1.1320	1.105	0.6659	1.56	0.2315	4.0	0.0315
1.025	1.1187	1.110	0.6519	1.58	0.2246	4.1	0.0299
1.026	1.1060	1.115	0.6387	1.60	0.2179	4.2	0.0285
1.027	1.0937	1.120	0.6260	1.62	0.2116	4.3	0.0272
1.028	1.0819	1.125	0.6139	1.64	0.2056	4.4	0.0259
1.029	1.0706	1.130	0.6025	1.66	0.1999	4.5	0.0248
1.030	1.0596	1.135	0.5913	1.68	0.1944	4.6	0.0237
1.031	1.0490	1.140	0.5808	1.70	0.1892	4.7	0.0227
1.032	1.0387	1.145	0.5707	1.72	0.1842	4.8	0.0218
1.033	1.0288	1.150	0.5608	1.74	0.1794	4.9	0.0209
1.034	1.0191	1.155	0.5514	1.76	0.1748	5.0	0.0201
1.035	1.0098	1.160	0.5423	1.78	0.1704	5.5	0.0166
1.036	1.0007	1.165	0.5335	1.80	0.1662	6.0	0.0139
1.037	0.9919	1.170	0.5251	1.82	0.1621	6.5	0.0118
1.038	0.9834	1.175	0.5169	1.84	0.1582	7.0	0.0102
1.039	0.9750	1.180	0.5090	1.86	0.1545	7.5	0.0089
1.040	0.9669	1.185	0.5014	1.88	0.1509	8.0	0.0077
1.041	0.9590	1.190	0.4939	1.90	0.1474	8.5	0.0069
1.042	0.9513	1.195	0.4868	1.92	0.1440	9.0	0.0062
1.043	0.9438	1.200	0.4798	1.94	0.1408	9.5	0.0055
1.044	0.9364	1.21	0.4664	1.96	0.1377	10.0	0.0050
1.045	0.9293	1.22	0.4538	1.98	0.1347	12.0	0.0035
1.046	0.9223	1.23	0.4419	2.00	0.1318	15.0	0.0022
1.047	0.9154	1.24	0.4306	2.05	0.1249	20.0	0.0013
1.048	0.9087	1.25	0.4198	2.10	0.1186	30.0	0.0006
1.049	0.9022	1.26	0.4096	2.15	0.1128	50.0	0.0002
1.050	0.8958	1.27	0.3998	2.20	0.1074	100.0	0.0001
1.052	0.8834	1.28	0.3905	2.25	0.1024	∞	0.0000

TABLE 601 (*Continued*)

BRESSE'S BACKWATER FUNCTION

Part 2 For *M2, M3,* and *S3* curves				Part 3 For *A2* and *A3* curves			
z	$\Phi[z]$	z	$\Phi[z]$	z	$\Phi[z]$	z	$\Phi[z]$
0.00	0.0000	0.935	1.3744	−0.00	1.2092	−1.50	0.1999
0.10	0.1000	0.940	1.4028	−0.10	1.1092	−1.55	0.1889
0.20	0.2004	0.945	1.4336	−0.15	1.0593	−1.60	0.1787
0.25	0.2510	0.950	1.4670	−0.20	1.0096	−1.65	0.1692
0.30	0.3021	0.952	1.4813	−0.25	0.9603	−1.70	0.1605
0.35	0.3538	0.954	1.4962	−0.30	0.9112	−1.75	0.1523
0.40	0.4066	0.956	1.5117	−0.35	0.8629	−1.80	0.1447
0.45	0.4608	0.958	1.5279	−0.40	0.8154	−1.85	0.1377
0.50	0.5168	0.960	1.5448	−0.45	0.7689	−1.90	0.1311
0.52	0.5399	0.962	1.5626	−0.50	0.7238	−1.95	0.1249
0.54	0.5634	0.964	1.5813	−0.55	0.6801	−2.0	0.1192
0.56	0.5874	0.966	1.6011	−0.60	0.6381	−2.1	0.1088
0.58	0.6120	0.968	1.6220	−0.65	0.5979	−2.2	0.0996
0.60	0.6371	0.970	1.6442	−0.70	0.5597	−2.3	0.0916
0.62	0.6630	0.971	1.6558	−0.75	0.5234	−2.4	0.0845
0.64	0.6897	0.972	1.6678	−0.80	0.4894	−2.5	0.0780
0.66	0.7173	0.973	1.6803	−0.85	0.4574	−2.6	0.0723
0.68	0.7459	0.974	1.6932	−0.90	0.4274	−2.7	0.0672
0.70	0.7757	0.975	1.7066	−0.95	0.3995	−2.8	0.0626
0.71	0.7910	0.976	1.7206	−1.00	0.3736	−2.9	0.0585
0.72	0.8068	0.977	1.7351	−1.02	0.3637	−3.0	0.0548
0.73	0.8230	0.978	1.7503	−1.04	0.3541	−3.2	0.0482
0.74	0.8396	0.979	1.7661	−1.06	0.3449	−3.4	0.0428
0.75	0.8566	0.980	1.7827	−1.08	0.3359	−3.6	0.0383
0.76	0.8742	0.981	1.8001	−1.10	0.3272	−3.8	0.0344
0.77	0.8923	0.982	1.8185	−1.12	0.3187	−4.0	0.0311
0.78	0.9110	0.983	1.8379	−1.14	0.3105	−4.2	0.0282
0.79	0.9304	0.984	1.8584	−1.16	0.3026	−4.4	0.0257
0.80	0.9505	0.985	1.8803	−1.18	0.2949	−4.6	0.0235
0.81	0.9714	0.986	1.9036	−1.20	0.2875	−4.8	0.0216
0.82	0.9932	0.987	1.9287	−1.22	0.2802	−5.0	0.0199
0.83	1.0160	0.988	1.9557	−1.24	0.2733	−5.5	0.0165
0.84	1.0399	0.989	1.9850	−1.26	0.2665	−6.0	0.0139
0.85	1.0651	0.990	2.0171	−1.28	0.2599	−6.5	0.0118
0.86	1.0918	0.991	2.0526	−1.30	0.2536	−7.0	0.0102
0.87	1.1202	0.992	2.0922	−1.32	0.2474	−8.0	0.0078
0.88	1.1505	0.993	2.1370	−1.34	0.2414	−9.0	0.0062
0.89	1.1831	0.994	2.1887	−1.36	0.2357	−10.0	0.0050
0.900	1.2184	0.995	2.2498	−1.38	0.2301	−12.0	0.0035
0.905	1.2373	0.996	2.3246	−1.40	0.2246	−15.0	0.0022
0.910	1.2571	0.997	2.4208	−1.42	0.2194	−20.0	0.0013
0.915	1.2779	0.998	2.5563	−1.44	0.2143	−30.0	0.0006
0.920	1.2999	0.999	2.7877	−1.46	0.2093	−50.0	0.0002
0.925	1.3232	1.000	∞	−1.48	0.2045	−∞	0.0000
0.930	1.3479						

be verified by inspection of equation (607). If y equals y_n, dy/dx must equal zero unless y_n is equal to y_c. This means that except in channels having critical slope, all curves must approach the normal depth line asymptotically. If y equals y_c, dy/dx must be infinite unless y_c is

equal to y_n. This means that except in channels having critical slope, all curves must cross the critical depth vertically. If y equals zero, dy/dx is a positive quantity which is equal to S_0 if y_c equals y_n. This means that the curves which approach the bottom intersect it at a definite angle. If y approaches infinity, dy/dx approaches S_0, which means that the water surface becomes horizontal as the depth of flow becomes very great. Finally, when y_n equals y_c, dy/dx must equal S_0 unless y equals y_n, when it may equal zero.

An *M1* curve is produced when the lower end of a long flume having a mild grade is submerged in a reservoir to a greater depth than the normal depth of flow in the flume.

An *M2* curve results when the bottom of the flume at its lower end is submerged in the reservoir to a depth less than the normal depth. If the depth of submergence is greater than the critical depth, then as much of the *M2* curve will form as lies above the water surface in the reservoir. If the amount of submergence is less than the critical depth the *M2* curve should end abruptly with its lower end tangent to a vertical line and at a height equal to the critical depth. Actually, vertical components of velocity will become an appreciable factor, and near the end it will merge into the local phenomenon known as the drop-off.

The *M3* curve is very peculiar in that its lower left-hand end starts from the bottom of the channel at an acute angle and its upper right-hand end terminates abruptly, tangent to a vertical line. Thus the curve is restricted in length to definite limits in both directions. On this account it can exist only under favorable circumstances.

For example, if water issued at high velocity from a reservoir through a submerged gate opening, it could follow this curve, provided that the water could be conducted away by some change in channel conditions, before the right-hand end of the curve should be reached. Such a change might be a sufficient increase in the grade of the channel. The curve could also be terminated abruptly at the right-hand end by a hydraulic jump. The location of the jump may be found by the method described in Chapter III, for rectangular channels, or that of Chapter IV, for other channels. Still another method of terminating the *M3* curve would be by one of the methods of changing from low stage to high stage without the hydraulic jump, as described in Chapter II.

The higher the initial velocity of the issuing water, the farther down towards the left the *M3* curve would begin. At the end of the curve, where it intersects the bottom of the channel, the velocity of the water would be infinite, so this point represents a limit that could never be reached physically.

Cases *S1*, *S2*, and *S3* correspond somewhat to cases *M1*, *M2*, and *M3*, respectively. It will be noticed, however, that some of the slopes and directions are reversed. The *M1*, *M2*, and *M3* curves change into *S1*, *S2*, and *S3* when the grade is increased until the normal depth is less than the critical depth. The same part of the table from which the *M1* curve is calculated gives *S1* and *S2*. The *M2*, *M3*, and *S3* curves are obtained from another part of the table.

Cases *C1* and *C3*, formed of straight lines, represent the limiting conditions between the *M1*, *M2*, and *M3* curves, on the one hand, and the *S1*, *S2*, and *S3* curves, on the other. The condition necessary for the formation of the *C1* and *C3* curves is that the slope be just sufficient to compensate for friction when the normal depth and critical depth become equal, a condition seldom encountered because of the delicate balance of slope and roughness required.

Cases *H2* and *H3* represent the limiting shapes approached by the *M2* and *M3* curves as the slope of the channel bottom approaches a true level. Under these circumstances the normal depth, y_n, has become infinite. Making use of the relations developed previously, equation (606) simplifies to

$$dx = \frac{C^2}{g}\left(1 - \frac{y^3}{y_c^{\,3}}\right) dy$$

The general integral of this is

$$x = \frac{C^2}{g}\left(y - \frac{y^4}{4y_c^{\,3}}\right) + \text{a constant} \qquad [609]$$

When the bottom slopes upward, the slope may be considered negative, y_n becomes negative, and hence z is negative. In using equation (608) with Part 3 of Table 601, care must be taken to see that none of these negative signs is overlooked.

Point of control. In deriving Bresse's equation for the backwater curves, no mention was made of the constant of integration. It was omitted because there is no advantage in fixing the mathematical origin of the x's at any particular point along the curve; the origin might as well have whatever arbitrary location results from using the equation in its most convenient form, without any constant of integration. The location of the curve along the stream is not arbitrary, however. Its position is fixed by a point of control at one end or the other of the backwater curve. For example, the longitudinal position of the *M1* curve is fixed by the requirement that the right-hand branch of the curve must meet the water surface in the reservoir into which the stream is discharging.

In general, if the flow follows any of the backwater curves, it must be because of some departure from the conditions that sustain uniform flow at the normal depth. There must be some change, such as a dam, a constriction, or a change of grade, to cause the flow to follow one of the curves rather than to continue parallel to the bottom of the channel. Imagine the change to be suddenly introduced into the uniform channel. A disturbance of the water surface will result, starting waves. If the velocity of the uniform flow is higher than the critical, any small waves that form will be swept downstream by the current. Large waves will cause a hydraulic jump, which may travel upstream for a short distance, but which must become stationary as soon as the point is reached where the downstream depth is sequent to the depth of the uniform flow.

If the velocity is less than critical, however, the waves will progress upstream without causing a jump, and the effect of the dam or constriction will extend upstream a considerable distance.

Thus it is seen that the control is downstream when the depth is greater than critical, and upstream when the depth is less than the critical. Experiments indicate that this criterion holds even when the depth is very near the critical. For example, submergence has no effect on the discharge of a broad-crested weir until the tailwater has reached the level of the critical depth over the crest.

Selection of Chezy's *C*. In using equations (608) or (609), the *variation* of the friction loss throughout the length of the backwater curve is evaluated by means of Chezy's formula. The error from this source can be minimized by using a more accurate friction formula to evaluate the *average* rate of loss. This is accomplished by choosing a value of C that will make Chezy's formula agree with Manning's (or Kutter's) at an intermediate depth. This value of C is then used throughout the range of the backwater curve. The intermediate depth used for determining C can be the arithmetical average of the depths at each end of the range, or it can be selected on the side of the mean in which the greater portion of the backwater curve will lie. The effect of convergence or divergence of the flow upon the friction loss can also be allowed for in this manner.

By Kutter's formula, C is a function of the slope. Care must be taken not to use the bottom slope, but rather the slope that would sustain uniform flow at the selected depth in the given channel.

The value of C determined upon should be used throughout the entire length of the curve. To break the curve into short lengths and use a different value of C in each may seem to offer an improvement in accuracy. This procedure is not justified, however, for the convenience of Bresse's method is lost, and the errors resulting from the assumption of

an infinitely wide rectangular cross section are still present. In some common cases the errors due to this assumption and to using a constant value of C tend to offset each other. If more accuracy is desired than can be obtained by Bresse's method with C constant, the methods described in later chapters should be used.

Flow in precipitous channels. In extremely steep channels, such as those which are sometimes built in mountainous country, the condition of uniform flow at normal depth is not stable. Instead, the water tends to flow in a series of pulsations, "roll waves," or "slugs," with the deepest part much deeper than the depth which would be indicated by the standard formulas for uniform flow. This may easily result in overtopping, which is always undesirable, and which may be dangerous where ground slopes are steep. Although no complete analysis of the phenomenon has been made which will allow the shape and velocity of the waves to be predicted, something is known of the conditions under which they will form. If the slope is long enough and steep enough, the roll waves will form without any perceptible disturbance to start them. They will start much sooner, however, and may build up to maximum size in a fairly short channel if there is some initial disturbance such as a side obstruction or irregularity near the upper end of the channel, or a train of waves in the reservoir at the head of the channel.

A value for the minimum slope that will sustain pulsating flow of this type has been obtained by Jeffreys and by Thomas, who used different methods, but who both assumed that Chezy's formula could be used to evaluate the effect of friction, and that the channel had an infinitely wide rectangular shape.[3,4] They obtain the same result, which is, in our notation, $4g/C^2$, or four times the ordinary critical slope. Using Manning's formula to evaluate friction, and assuming an infinitely wide rectangular channel, Keulegan and Patterson obtain as a result a slope of $2\frac{1}{4}$ times the ordinary critical slope.[5] Observations, however, indicate that serious roll-wave disturbances seldom form except on steeper slopes than indicated by either of these criteria. This may be because a very great length would be required for the waves to build up, at slopes near the minimum, or because the neglected frictional effect of the channel sides is appreciable.

Backwater curves, as described in this book, cannot exist in pre-

[3] "The Flow of Water in an Inclined Channel of Rectangular Section," by Harold Jeffreys, *Phil. Mag.*, [6] 49 (1925), 793.

[4] "The Propagation of Waves in Steep Prismatic Channels," by Harold A. Thomas, *Proceedings of Hydraulics Conference, Studies in Engineering, Bulletin* 20, State University of Iowa, Iowa City.

[5] "A Criterion for Instability of Flow in Steep Channels," by G. H. Keulegan and G. W. Patterson, *Transactions American Geophysical Union*, 1940, Part II, p. 594.

cipitous channels except when the distance is so short that the roll waves do not have time to develop.

Illustrative Example

A river has a uniform slope of $S_0 = 0.0004$ and is at a flood stage of $y_n = 10$ feet. Determine the backwater curve produced by a dam at which $y = 25$ feet. Use $n = 0.022$.

From inspection of the given data, it seems most likely that the curve will be case *M1*. To verify this, compute the value of y_c.

$$V = \frac{1.486}{0.022} \times 10^{2/3} \times 0.0004^{1/2} = 6.3 \text{ ft./sec.}$$

$$Q \text{ per foot} = 63 \text{ c.f.s.}$$

$$y_c = \sqrt[3]{\frac{Q^2}{g}} = \sqrt[3]{\frac{63 \times 63}{32.2}} = 5.0 \text{ ft.}$$

Since $y_c < y_n < y$, the curve is an *M1* curve, and is given by equation (608)

$$x = \frac{y_n}{S_0} z - y_n \left(\frac{1}{S_0} - \frac{C^2}{g} \right) \Phi[z]$$

The value of C corresponding to the depth of 10 feet is

$$C = \frac{1.486}{0.022} \times 10^{1/6} = 99$$

by Manning's formula. Similarly, C is 115 at the depth of 25 feet. Since the greater part of the curve lies in the range of the shallower depths, and since, furthermore, the flow is expanding and thus is subject to greater loss than uniform flow at the same depth, let us use $C = 105$ in applying Bresse's formula. Substituting the given values, with distances in miles, equation (608) becomes

$$x = 4.74z - 4.09\Phi[z]$$

The curve can be plotted from data in the following table:

$z = \frac{y}{y_n}$	$\Phi[z]$ from Table 601	$4.74z$	$4.09\Phi[z]$	x	Distance from dam, in miles
2.5	0.0821	11.83	0.34	11.49	0
2.3	0.0978	10.89	0.40	10.49	1.0
2.1	0.1186	9.94	0.49	9.45	2.04
1.9	0.1474	9.00	0.60	8.40	3.11
1.7	0.1892	8.05	0.77	7.28	4.21
1.5	0.2548	7.10	1.04	6.06	5.43
1.3	0.3731	6.16	1.53	4.63	6.86
1.1	0.6806	5.21	2.78	+2.43	9.06
1.01	1.4192	4.78	5.80	−1.02	12.51

The computations in the table are explained by the column headings. The column headed " x " is obtained directly as indicated by the equation above, and represents the distance from the arbitrary origin which happens to be 11.49 miles upstream from the dam. A larger number of values of z could have been included in the table if more points on the curve had been needed. On the other hand, as few values could have been used as desired. There is no necessity that the intervals between computed values be small.

PROBLEMS

603. Assume that the channel of the illustrative example has a sudden drop-off instead of a dam, all other conditions remaining the same. Determine the backwater curve, and plot it to an exaggerated vertical scale.

604. Water flows from under a gate in a wide rectangular channel. The bottom of the channel is level to a point 1,000 feet downstream from the gate, where it has a sudden drop-off. The bottom of the gate is 5 feet above the floor of the flume and the depth of the water upstream is 20 feet. Assuming that the depth at the vena contracta is 3.9 feet and that $n = 0.013$, determine the water-surface profile from the gate to the drop-off.

605. Water spills over a levee by-pass and flows down a smooth concrete apron built on a slope of 12 horizontal to 1 vertical. What will be the depth of the water 100 feet downstream from the crest, if the head on the crest is 3 feet, and the friction losses up to the crest may be neglected?

CHAPTER VII

BACKWATER CURVES IN UNIFORM CHANNELS

Backwater curves in uniform channels of standard rectangular or trapezoidal cross section may be computed by means of Bresse's function if the cross section is comparatively shallow and wide and the curve does not cover too great a range of depths. Otherwise, profiles computed with the aid of Bresse's function may differ appreciably from profiles determined by field measurements. The experiments reported by Nagaho Mononobe show that for close agreement between the computed and the observed curves in channels of narrow deep cross section, a better friction formula than Chezy's must be used, and the actual shape of the channel must be considered.[1]

The method to be used, if Bresse's is judged to be unsatisfactory, will depend upon the type of problem to be solved. When only a few curves have to be computed and elevations are needed all along the length of each curve, the step method for uniform channels described in Chapter IX is most convenient. For another type of problem, such as the determination of the flow in a channel connecting two reservoirs, a large number of curves may be needed, and the labor of computation, if the step method is used, may become excessive. The elevations determined for the intermediate steps are not necessary except in that they must be computed to obtain the desired elevation at the far end of the channel. The graphical method for uniform channels avoids this difficulty, and is most convenient for the solution of problems of this type.

Graphical method for computing backwater curves in uniform channels. In the derivation of Bresse's function, it was assumed that the rate of loss of energy through friction, in non-uniform flow, is the same as in uniform flow at the same velocity and depth. This assumption is again necessary, but the friction loss will be evaluated by a modern type of formula. The actual shape of the channel cross section will be taken into account, in its effect on both friction loss and velocity head changes. The friction slope, or rate of loss of head through friction,

[1] "Backwater and Drop-down Curves for Uniform Channels," by Nagaho Mononobe, *Trans. Am. Soc. Civil Eng.*, v. 103 (1938), p. 950.

is given by either

$$S_f = \left(\frac{nV}{1.486R^{2/3}}\right)^2 = \frac{n^2Q^2}{2.21A^2R^{4/3}} \qquad [701a]$$

or

$$S_f = \left(\frac{V}{C\sqrt{R}}\right)^2 = \frac{Q^2}{C^2A^2R} \qquad [701b]$$

depending upon whether the Manning or Kutter formula is to be used. By the Manning formula, S_f is a function of the depth alone for a given discharge in a given channel. This is also true when the Kutter formula is used, for though C is a function of the slope, the correct slope to use is not the variable water-surface slope, nor the slope of the channel bottom, but the friction slope S_f.

The rate of change of velocity head is

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dx}\frac{Q^2}{2gA^2} = \frac{-Q^2}{gA^3}\frac{dA}{dx} = \frac{-Q^2}{gA^3}\frac{dA}{dy}\frac{dy}{dx} = \frac{-Q^2T}{gA^3}\frac{dy}{dx} \qquad [702]$$

In this expression T is the surface width, A is the area of flow, dy is an increment of the variable depth of flow corresponding to dx, an elementary distance along the stream, and the other factors have their customary significance, so that the multiplier of dy/dx is seen to be a function of the depth alone for a given discharge in a given channel.

Equations (701) and (702) may now be substituted in the general differential equation developed in Chapter VI,

$$S_0 - \frac{dy}{dx} = \frac{d}{dx}\left(\frac{V^2}{2g}\right) + S_f$$

giving, after separation of the variables,

$$dx = \frac{1 - \dfrac{Q^2T}{gA^3}}{S_0 - S_f}\,dy \qquad [703a]$$

This equation is of the form

$$dx = f(y)\,dy \qquad [703b]$$

The function $f(y)$ is usually of a form that is difficult if not impossible to integrate by the methods of ordinary calculus, so that graphical integration is to be preferred. Values of $f(y)$ are computed corresponding to values of y covering the range of depths of the backwater curve. Values of $f(y)$ are then plotted against y on Cartesian coordinates, and a smooth curve drawn through the points. The area under the curve $f(y)$ between y_1 and y_2 is the distance along the stream from where the depth is y_1 to where it is y_2.

By using empirical equations to represent the variable factors making up $f(y)$, the integration may be accomplished. Mononobe gives tables for complete evaluation of the integral.[2] Unfortunately his method is marred by inadmissible approximations. Bakhmeteff gives tables which completely evaluate the integral only when the effect of velocity head changes is neglected.[3] Though his method and tables are accurate, the correction which must be made to take the velocity head changes into account destroys the convenience of the method except when the effect of the velocity head changes is insignificant.

Illustrative Example

A trapezoidal canal with bottom width of 100 feet, side slopes of 1 on 1, and a gradient S_0 of 0.0004 delivers a discharge of 6220 c.f.s. to a large reservoir. Determine the depth of flow at the entrance of the canal, 7.0 miles upstream from the reservoir, if the depth of flow at its lower end is 25 feet. Use $n = 0.022$ throughout.

The data are chosen to provide a comparison with the results of the illustrative example solved by Bresse's method. In that example the channel was considered to have an infinitely wide rectangular cross section. The normal depth of flow, 10 feet, is the same in each case, and the curve is an *M1* curve, as before. The computations for the graph of $f(y)$ are made in Table 701, and the results are plotted in Fig. 701(a). The values of depth chosen for the computations of Table 701 are selected so that the points for plotting the graph will be more closely spaced where the curvature is small. Had points closer to the normal depth been needed, smaller increments of depth would have been necessary for the interval between 10 and 11 feet of depth.

The depth at the upper end of the canal is found by determining a value of y such that the area under the curve between $y = y$ and $y = 25$ is $5{,}280 \times 7 = 36{,}960$ feet. This was done by first planimetering the area from $y = 25$ to $y = 13$, which was found to be 34,900 feet. Then the position of the line to enclose the remaining area on the diagram was estimated, and after checking, was found to be $y = 12.55$, which is, then, the required depth of flow 7 miles upstream from the lower end of the channel.

The depth at the same point, as determined by interpolating in the results of the example solved by Bresse's method, is 12.87, a difference of less than 3 per cent. The close correspondence is due to the fact that the cross section is comparatively broad and shallow.

When more than one depth is required it is advantageous to plot a mass curve to give the area under $f(y)$. The mass curve for the curve of Fig. 701(a) is shown in Fig. 701(b), which shows directly that the depth is 12.55 feet 7 miles upstream from where it is 25 feet. The answer to any other such problem within the range of depths covered, and for the same discharge, may be read

[2] Ibid.

[3] *Hydraulics of Open Channels*, Boris A. Bakhmeteff, McGraw-Hill, 1932.

TABLE 701

COMPUTATIONS FOR EXAMPLE ILLUSTRATING GRAPHICAL METHOD

1	2	3	4	5	6	7	8	9	10	11
Depth	Surface Width	Area	Wetted Perimeter	Hydraulic Radius			Numerator	S_f	Denominator	$f(y)$
y	$T = 100+2y$	$A = 100y+y^2$	$P = 100+2\sqrt{2}y$	$R = \frac{A}{P}$	$R^{4/3}$	$\frac{Q^2T}{gA^3}$	$1 - \frac{Q^2T}{gA^3}$	$\frac{n^2Q^2}{1.486^2 A^2 R^{4/3}}$	$S_0 - S_f$	8/10
11	122	1221	131.1	9.31	19.6	0.0804	.9196	.000290	.000110	8360
12	124	1344	134.0	10.02	21.6	0.0614	.9386	.000217	.000183	5130
13	126	1469	136.8	10.75	23.6	0.0479	.9521	.000167	.000233	4090
14	128	1596	139.6	11.42	25.7	0.0377	.9623	.000129	.000271	3550
16	132	1856	145.3	12.78	29.8	0.0248	.9752	.000083	.000317	3080
18	136	2124	151.0	14.08	34.0	0.0170	.9830	.000055	.000345	2850
21	142	2541	159.4	15.93	40.0	0.0104	.9896	.000033	.000367	2700
25	150	3125	170.7	18.31	48.2	0.0059	.9941	.000018	.000382	2600

$n = 0.022$ $\quad S_0 = 0.0004$ $\quad Q = 6220$ $\quad y_n = 10$ ft.

from Fig. 701(*b*). Curves for other values of the discharge can be plotted on the same graph. In obtaining $f(y)$ for other discharges, only columns 7 to 11 of Table 701 need to be recomputed, for the values in columns 7 and 9 will be proportional to the square of the value of the discharge.

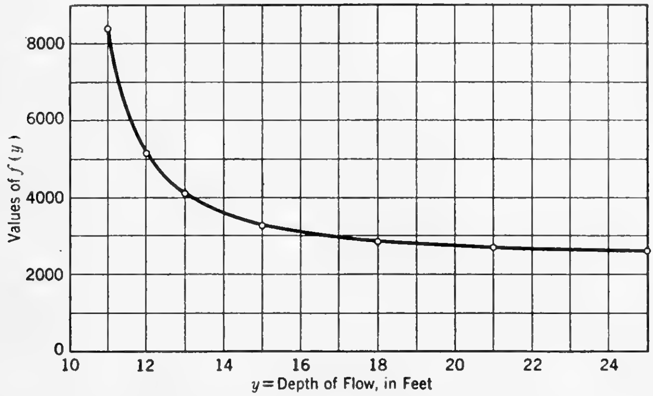


FIG. 701(*a*). Plot of $f(y)$ Computed in Table 701.
Areas under this curve represent distances along the channel.

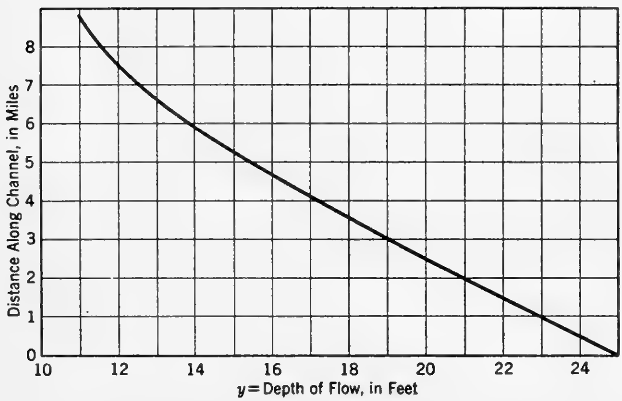


FIG. 701(*b*). Summation of Areas under the Curve of Fig. 701(*a*).

PROBLEM

701. Assume that the canal of the illustrative example has a sudden drop-off at the lower end, all other data remaining the same. Determine the backwater curve, and compare it with that of problem 603.

Backwater curves in horizontal uniform channels: preliminary. For the *H2* and *H3* backwater curves, which are the important special cases for which the channel bottom is horizontal, a different treatment is

desirable. The first step is to determine the coefficients and exponents in the following empirical equations:

$$\frac{A^3}{T} = c_a y^a \tag{704}$$

and

$$A^2C^2R = c_b y^b \quad \text{or} \quad \frac{\overline{1.486}^2 A^2 R^{4/3}}{n^2} = c_b y^b \tag{705}$$

in which A, T, and R represent the area, surface width, and hydraulic radius when the depth of flow is y; C and n are the Kutter and Manning coefficients most appropriate at the depth y; and c_a, c_b, a, and b are the empirical coefficients and exponents to be determined. The choice of equations (705) depends upon whether the Kutter or the Manning formula is to be used in evaluating the effect of friction.

Bakhmeteff calls the function $AC\sqrt{R}$ or $\dfrac{1.486}{n} AR^{2/3}$ the "conveyance" of the channel at depth y. This quantity has only to be multiplied by the square root of the slope to give the discharge for uniform flow at the given depth. Bakhmeteff discovered that for both artificial and natural channels the square of this function is almost invariably susceptible of accurate approximation by a monomial exponential function of the depth.[4]

For rectangular or triangular channels, the coefficient and exponent in equation (704) may be determined by simple algebra. For other channels, corresponding values of A^3/T and y over the range of depths of the backwater curve are plotted logarithmically, with values of $\log y$ as abscissas. A straight line is drawn fitting the points as well as possible. Measurement of its slope will give the numerical value of the exponent a. The coefficient c_a is obtained by substituting a pair of simultaneous values for a point on the line, or by extending the line to its intersection with the line y equals unity. A similar procedure is employed for the determination of the coefficient c_b and the exponent b.

Another method for determining a and b is simpler, and often just as accurate. Let the subscripts 1 and 2 refer to values of the variables at the lower and upper end of the range of depths. Then

$$a = \frac{\log (A_2{}^3/T_2) - \log (A_1{}^3/T_1)}{\log y_2 - \log y_1}$$

and

$$b = \frac{\log (A_2{}^2R_2{}^{4/3}) - \log (A_1{}^2R_1{}^{4/3})}{\log y_2 - \log y_1}$$

[4] Ibid.

Determining a and b by this procedure corresponds to fitting the straight line through the initial and final points on the logarithmic plot. It has the disadvantage of not revealing the degree of approximation for the intermediate points, but it is much quicker than the graphical method, and can safely be used when experience indicates that the logarithmic plot will differ but little from a straight line. After a and b have been determined by this method, the coefficients c_a and c_b are found by substitution of a pair of simultaneous values, preferably for an intermediate point.

A suggestion which seems to offer an obvious improvement in computation procedure is that the desired exponents could be obtained by simple mathematical processes from empirical exponential relations for the surface width and the wetted perimeter. The relation for the surface width would only have to be integrated to obtain the relation for the area, which could be divided by the relation for the wetted perimeter to obtain an expression for the hydraulic radius, and so on. These operations could be performed more easily upon the simple empirical equations than upon the original data. Unfortunately, exponential values obtained by this process are likely to be very inaccurate. The logarithmic plots of the surface width and the wetted perimeter do not approximate straight lines very well, and the errors produced by performing mathematical operations upon empirical relationships are likely to be surprisingly large. On the other hand, the data for equations (704) and (705) can almost always be closely approximated by a straight line on the ogarithmic plot. The errors introduced in the final integration of the backwater function may be kept to a minimum by determining the necessary empirical relations directly in the form of equations (704) and (705).

Illustrative Problem

Determine values of the coefficients and exponents in equations (704) and (705) for a trapezoidal channel with bottom width of 100 feet and side slopes cf 1 on 1, to apply over a range of depths between 10 and 25 feet. Let $n = 0.022$.

The computations are based on the values for the same cross section which were computed in Table 701.

Depth	A^3/T	$\frac{\overline{1.486}^2 A^2 R^{4/3}}{n^2}$
10	1.11×10^7	9.68×10^{10}
12	1.96	1.78×10^{11}
14	3.17	2.98
16	4.84	4.68
18	7.06	7.01
21	11.56	11.79
25	20.35	21.45

From the logarithmic plots, shown on Fig. 702, the empirical equations representing the straight lines are found to be

$$\frac{A^3}{T} = 7340y^{3.18}$$

and

$$\frac{\overline{1.486}^2 A^2 R^{4/3}}{n^2} = 42{,}300{,}000y^{3.36}$$

Since in this example n did not vary with the depth, it is obvious that the multiplier $\overline{1.486}^2/n^2$ could have been omitted in determining the exponent b. It would have to be included in computing the value of c_b, however.

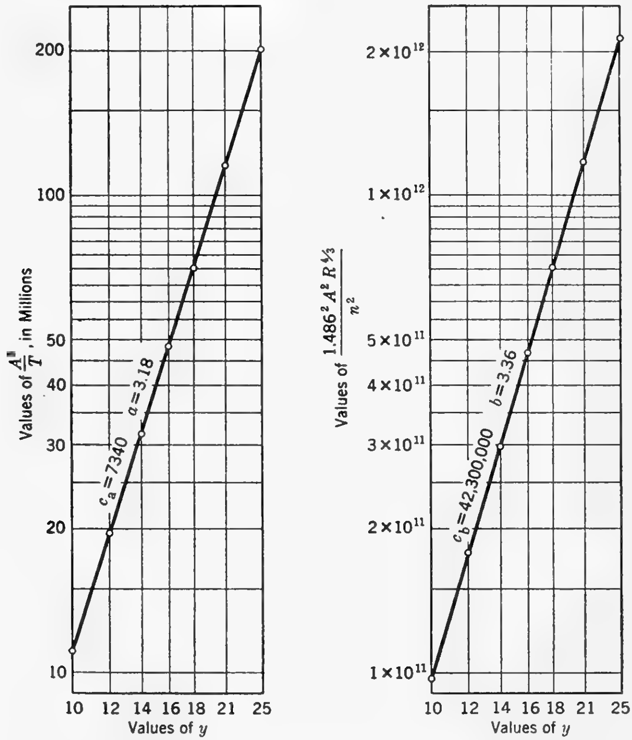


FIG. 702. Logarithmic Plot for the Determination of the Coefficients and Exponents in Empirical Equations (704) and (705).

By the other method

$$a = \frac{\log 20.35 - \log 1.11}{\log 25 - \log 10} = 3.18$$

and

$$b = \frac{\log 214.5 - \log 9.68}{\log 25 - \log 10} = 3.38$$

The slightly different result for the exponent b will require a different value of the corresponding coefficient:

$$c_b = \frac{1.78 \times 10^{11}}{12^{3.38}} = 40{,}100{,}000$$

In this particular problem, not one of the three empirical equations developed gives results differing more than 2 per cent from the values to which they were fitted.

PROBLEM

702. The backwater curve in a circular conduit 10 feet in diameter lies between the depths of 2 and 6 feet. Determine appropriate values of the exponents and coefficients in equations (704) and (705). Use $n = 0.012$.

Backwater curves in horizontal uniform channels: integration. To proceed, now, with the integration of the backwater curves in horizontal channels, the differential equation takes the special form

$$S_w = -\frac{dy}{dx} = S_f + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \qquad [706]$$

in which

$$S_f = \frac{V^2}{C^2R} = \frac{Q^2}{A^2C^2R} = \frac{Q^2}{c_by^b}$$

and

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{-Q^2T}{gA^3}\frac{dy}{dx} = \frac{-Q^2}{gc_ay^a}\frac{dy}{dx}$$

so that

$$-\frac{dy}{dx} = \frac{Q^2}{c_by^b} - \frac{Q^2}{gc_ay^a}\frac{dy}{dx}$$

Separating the variables

$$-dx = \left(\frac{c_by^b}{Q^2} - \frac{c_b}{gc_a}\,y^{b-a}\right)dy$$

Integrating

$$x_1 - x_2 = \left[\frac{c_b}{Q^2(b+1)}\,y^{b+1} - \frac{c_b}{gc_a(b+1-a)}\,y^{b+1-a}\right]_{y_1}^{y_2} \qquad [707]$$

Equation (707) facilitates the computation of the cases of backwater curves in uniform channels with horizontal grade line.

PROBLEM

703. Solve problem 604, taking into account the shape of the channel, a rectangular concrete flume 20 feet wide.

CHAPTER VIII

ANALYSIS OF FLOW PROBLEMS

It has been shown in the preceding chapters that the water surface, for flow in a channel of non-varying cross section, may follow any one of a number of different backwater curves. The water may also flow uniformly at the normal depth, or it may go through a hydraulic jump. Examples have been given of conditions under which each of the numerous curves may form. The problem encountered in practice, however, is different. Given a certain situation, such as a proposed flume layout, the engineer is asked to predict the profile of the water surface. The proposed layout may include a channel profile unlike any of the familiar examples, so that an analysis of the particular problem must be made.

For the purposes of preliminary design, it will often be sufficient to determine the general shape of the water-surface profile, with a few controlling dimensions. This information will serve later as the starting point of the detailed profile computations, if these are required for the final design. It is not intended to formulate a set procedure by means of which the water-surface curves may be determined for every possible open-channel flow problem; instead, certain typical problems will be worked out, which will serve to indicate the method of attack on problems that are more complicated in their detail.

Changes of grade in straight channels of non-varying cross section. Backwater curves form in uniform channels over changes of grade and near the points of entrance and discharge. We consider first the different possible water-surface profiles which are caused by changes in the bottom grade of the channel. There is assumed to be but a single change of grade; the slope on each side is continued constant for an infinite distance. This is equivalent to assuming that the next change of grade, or other condition disturbing the normal flow, is sufficiently far away to be of negligible effect. It also usually means that the effects of the change of grade do not reach to the intake of the channel. It will therefore be assumed, unless otherwise stated, that the discharge is determined by some remote condition, and is known.

The various cases for which sustained steady flow is possible under the assumptions made are shown in Fig. 801. In each, the first step

in the analysis of the flow profile is to compute the critical and normal depths for the given discharge. Since the cross section of the channel remains the same through the change of grade, the critical depth remains the same. The normal depth will be different on each side of the change of grade, and its value relative to the value of the critical depth furnishes the basis for classification.

The first case shown at Fig. 801 (*a*) illustrates the change from a mild slope to a flatter mild slope. The normal depth is greater for the flatter mild slope, and both normal depths are greater than the critical. An *M1* backwater curve forms above the mild slope at the left. It joins the greater normal depth above the break of grade. No curve may form above the flatter slope because (1) none of the backwater curves in channels of mild slope (Fig. 602) become tangent to the normal depth in the downstream direction, and (2) the flow is deeper than the critical throughout, so that the break in grade can affect the profile in the upstream direction only.

Figure 801 (*b*) shows a break in grade from a mild slope to a steeper mild slope. An *M2* curve forms upstream from the change of grade, and joins the normal depth line of the steeper slope. This is also true for Fig. 801 (*c*) where the change of grade is from mild slope to critical slope. Flow at or near the critical slope, however, is especially subject to chance fluctuations, so that the profile in Fig. 801 (*c*) cannot be predicted with as much assurance as that in Fig. 801 (*b*).

Figure 801 (*d*) shows a transition from a mild slope to a steep slope. An *M2* curve forms above the mild slope, and an *S2* curve forms above the steep slope. The flow passes through the critical depth over the change of grade. According to the ordinary theory, both of the backwater curves become vertical as they approach the critical depth. Actually, the profile does not cross the critical depth vertically, because of the influence of the vertical components of velocity, which are neglected in the theory of the backwater curves. For the same reason, the critical depth may not occur precisely above the change of grade. If it is necessary to know the exact shape of the profile in the immediate neighborhood of the break in grade, recourse may be had to model study. A short distance away on each side, slopes become flat and the backwater curves apply with good accuracy.

Figure 801 (*e*) shows a change in grade from critical slope to mild slope. A *C1* curve forms over the critical slope, connecting with the normal depth in the channel of mild slope. This represents a passing transition between the case of Fig. 801 (*a*), in which the backwater curve extends a long distance upstream, and that of Fig. 801 (*g*), in which the curve extends downstream (if the tailwater is shallow).

Figure 801 (*f*) shows the change from critical slope to steep slope. An *S2* curve forms over the steep slope, starting from the critical depth over the brink. Here again the vertical components of velocity become of importance; the sharp corner called for by the backwater curve theory would be rounded, the effect extending a short distance up and down stream.

Figure 801 (*g*) shows a change of grade from steep to mild. Here different profiles may form depending upon the relative steepness of the two grades. If the normal depth on the mild slope is comparatively small, the swiftly flowing stream on the steep slope will continue flowing at uniform depth right up to the change of grade, where an *M3* curve begins. This curve continues on down the mild slope, increasing in depth and decreasing in velocity until a hydraulic jump forms, after which flow continues at the normal depth on the mild slope. The location of the jump can be found by the method explained in Chapter III. As the mild slope becomes flatter, the normal depth on it increases, and the jump moves upstream. When the normal depth becomes comparatively large, the jump forms upstream from the change of grade, and is followed by an *S1* curve joining the downstream normal depth over the change of grade.

Figure 801 (*h*) shows a change of grade from steep to critical. Flow continues down the steep slope at its normal depth to the change of grade, where a *C3* curve forms, extending horizontally across to its intersection with the normal depth on the critical slope. This is another transition case, the existence of which depends upon a delicate balance between the roughness and slope of the downstream portion of the channel.

Figures 801 (*i*) and (*j*) show the two possibilities when there is a break in grade with steep slopes on each side. The flow is normal down to the change of grade, after which an *S3* curve or an *S2* curve forms, depending upon whether the downstream grade is flatter or steeper than the upstream grade.

Figures 801 (*k*), (*l*), and (*m*) show changes from adverse slope to mild, critical, and steep slopes, respectively. An *A2* curve forms over the adverse slope in each of them. This curve joins the normal depth line for the mild slope, but reaches the critical depth at the crest for the critical slope and the steep slope. An *S2* curve forms on the steep slope, while above the critical slope the flow is at the normal depth from very near the crest. The sharp intersection of the normal depth line and the *A2* curve would of course be rounded.

A point of particular interest about the profiles shown in Figs. 801 (*k*), (*l*), and (*m*), is that the discharge is not fixed by upstream

channel conditions, as for all the other cases, but by the level of the horizontal asymptote of the $A2$ curve. Computation of the discharge,

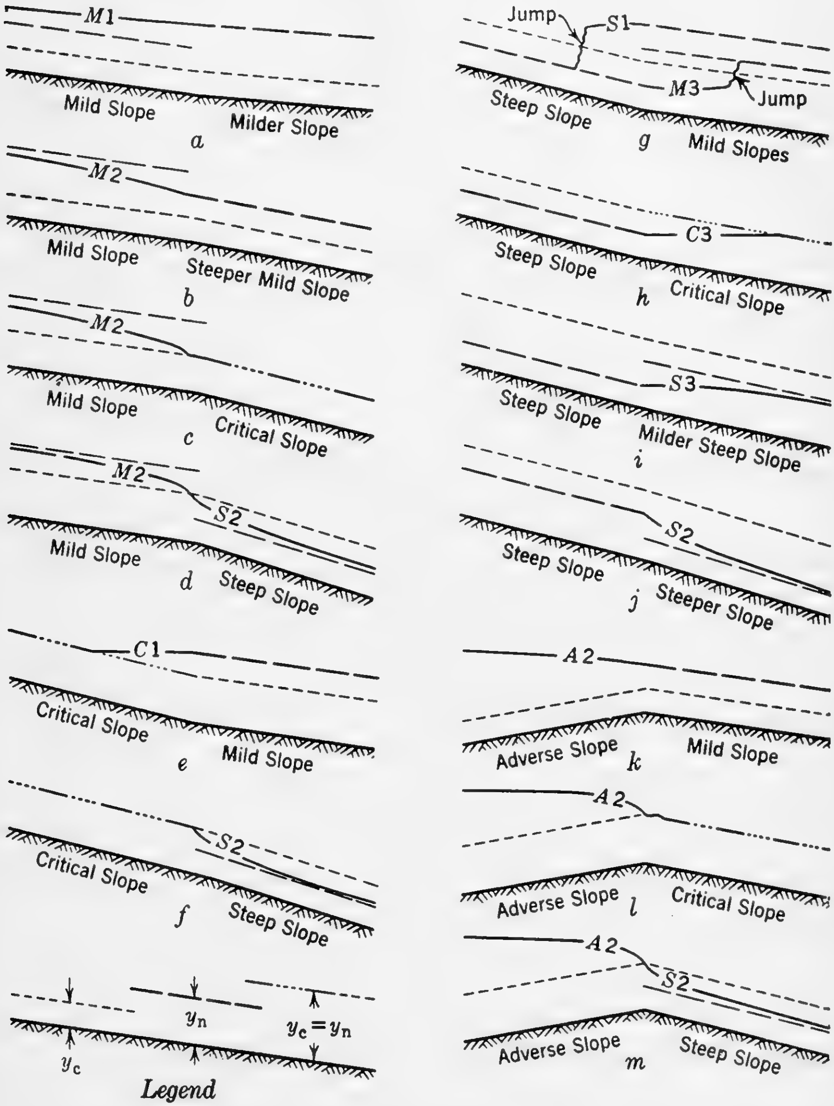


FIG. 801. Water-surface Profiles over a Break in Grade, for Steady Flow in a Long Prismatic Channel.

for a given elevation of the asymptote (or of the water level in a pool at the upstream end of the curve), is different for profile (k) than for

profiles (l) and (m). Let us consider profiles (l) and (m) first. An approximate value of the discharge can be computed on the basis of assumptions that there is no loss of energy up to the crest, and that flow over the crest is at the critical depth. This approximate value of the discharge may be used to compute the value of the normal depth on the downstream slope, to check whether the slope is actually critical or steep, as assumed. (Until the discharge is known, it may not be certain whether the slope is mild or steep.) Having tentatively determined that the flow is as shown in (l) or (m), and not (k), the $A2$ curve should be computed upstream to the source of supply at the upper end of the adverse slope. The pool level thus computed, which should include allowance for the velocity head at the entrance to the adverse slope, will be above the known pool elevation. If the difference is appreciable, a lower value should be assumed for the critical depth at the downstream end of the adverse slope, and the pool elevation recomputed. The computed pool elevations will lie on each side of the given pool elevation, if the second critical depth was chosen carefully, so that the discharge corresponding to the given pool elevation may be determined by interpolation.

To determine the discharge when the profile is as shown in Fig. 801(k), first find the value of the discharge corresponding to the normal depth for which the total head at the crest is equal to the height of the pool level above the crest, and also the value corresponding to another slightly smaller normal depth. Then proceed as before, figuring back to the pool level and interpolating to find the correct discharge. Other methods of computation may be used for these cases, but the procedure outlined here is as simple as any, and has definite advantages when the discharge must be known for a range of pool elevations.

Figure 801 shows all of the possibilities for steady flow over a break in grade in an infinitely long channel of constant cross section. No changes of grade to or from the horizontal are included, for steady flow would be impossible if the flow came from, or discharged into, an infinitely long channel with horizontal bottom.

The degree of certainty with which the profile to be expected under any given condition can be predicted, depends upon two factors: (1) accuracy of the evaluation of the friction losses, and (2) nearness of the depth of flow to the critical depth. Where the depth of flow is well above or well below the critical depth, the effect of inaccuracy in the evaluation of the friction losses is not great, but where the depth of flow is near the critical a small uncertainty in the friction loss corresponds to a large uncertainty in the depth of flow. For example, a slope estimated to be a steep slope, with depth of flow slightly less than the

critical, may actually be a mild slope, with depth of flow above the critical. A comparatively small change in the roughness coefficient may produce a marked change in the flow profile.

The vertical scales in all the profiles shown in Fig. 801 are enormously exaggerated. This makes the breaks in grade of the channel bottom look sharp, and perhaps in need of rounding. Actually, few such changes in grade need to be rounded. For high velocity flow crossing a brink or a crest, a vertical curve may be needed in order to prevent the formation of dangerously low pressures or excessive eddy disturbances in the lee of the corner. If a crest is rounded, the crest elevation is the actual elevation of the highest point on the curve, and not the intersection of the straight grade lines from each side. Vertical curves are sometimes used, though not needed for hydraulic reasons, in order to save on construction costs.

A treatment of all the possible cases of two breaks in grade sufficiently close together to prevent the establishment of normal flow in the stretch between them would be very lengthy, and, it is believed, unnecessary. The student who can sketch, without reference to notes, all of the profiles shown in Fig. 801 should be qualified to solve the more complicated profiles resulting from two or more changes of grade in the channel bottom.

PROBLEMS

801. A long portion of channel at mild slope is followed by a section at a still flatter slope, and then by a steep section. The cross section is the same throughout. Sketch the various possible water-surface profiles.

802. A long steep portion of a channel is followed by a horizontal section, and then a long section with mild slope. The cross section is constant. Sketch the various possible profiles.

803. A long rectangular concrete flume 12 feet wide has a change of grade from 0.004 to 0.001. If the flume is new and well finished, what will be the flow profile across the change of grade when the discharge is 300 c.f.s.? What will be the influence of increased roughness?

804. A channel is to be built through a ridge to serve as the spillway for a reservoir. In order to save rock excavation the channel bottom will be built on a rising grade of 2 per cent from the reservoir straight to the crest, a distance of 500 feet. For 200 feet past the crest, the channel bottom will have a steep grade down the hillside. What will be the discharge when the water level in the reservoir is 12 feet above the elevation of the crest, if the channel is 200 feet wide?

805. What would be the discharge, for the channel of problem 804, if the downward grade past the crest was constant at 0.003 for a great distance?

Uniform channel with lower end submerged in lake or reservoir. The different conditions which may result in the formation of backwater curves in a channel discharging into a pool are shown in Fig. 802.

If the slope is steep, a hydraulic jump may form, followed by an *S1* curve. For this curve to form as shown, the channel must extend out past the end of the hydraulic jump, and the side walls must be high enough to prevent water from flowing into the channel from the surrounding pool. If the depth of flow is at the critical, flow at the outlet is as shown for the critical slope, in Fig. 802, provided that the channel extends far enough out into the pool to insure that velocities at the end have been reduced enough to prevent any large-scale disturbance.

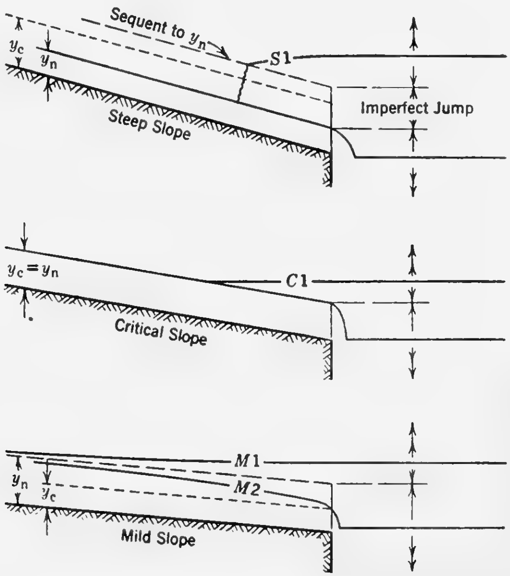


FIG. 802. Long Uniform Channel with Lower End Submerged in Reservoir or Lake.

Flow on the steep slope and on the critical slope in Fig. 802 is shown as uniform flow at the normal depth. The results will be nearly the same if there is a backwater curve on the slope which converges toward the normal depth as it approaches the pool level. Since the flow is at or below the critical depth, the discharge and flow down the slope are determined entirely by upstream conditions, and are not affected by the level of the water in the pool.

The pool level does affect the curve upstream if the channel has a mild slope. Two possibilities may be distinguished, as shown in Fig. 802. If the water level in the pool is higher than the level of the normal depth at the end of the channel, an *M1* curve will form; if lower, an *M2* curve will form. Under either circumstance the pool elevation affects the profile for some distance upstream, and thus may affect the

discharge. The distance upstream that the effect is appreciable depends upon the proportions of the channel, its roughness, and the pool level. It may vary from a few hundred feet to a number of miles. It should be noted that the velocity head of the stream at the end of the channel is lost, the energy going into eddies in the reservoir pool.

Entrance to a uniform channel. The discharge into a long uniform channel leading from a reservoir or lake depends upon the following factors, which are listed in order of importance.

Channel slope critical or steep:

1. head upon the crest.
2. shape of the entrance, contraction.
3. friction loss in the entrance.

Channel slope mild:

1. normal flow in the channel at total head equal to the head on the crest minus the
2. friction loss in the entrance.

Whether the slope is steep, critical, or mild depends, in turn, upon the discharge, so that we have a vicious circle. Problems are solved by the method used for computing flow from an adverse slope to a sustaining slope. Friction loss over the adverse slope is replaced by friction loss in the entrance section, which may be taken into account by applying an appropriate velocity coefficient, as is done for orifices and short tubes.

If the entrance to a channel of mild slope is not well rounded, eddies will develop, and the resulting energy loss will be comparatively high. A poorly rounded entrance to a channel of steep slope may cause contraction, with actual separation from the walls, and a corresponding decrease in the discharge. It is of interest to note that any instability in the flow at the entrance of channels with critical or steep slope is likely to cause disturbances similar to the inversions of a free jet, decreasing the expected capacity by encroaching upon the freeboard.

Channel connecting two reservoirs or lakes. Two large reservoirs or lakes are connected by a channel. The water level in each reservoir is subject to slow changes, and the flow in the connecting channel may be in either direction. It is desired to know, for any given combination of reservoir elevations, what the discharge in the channel will be. The rate of change of the lake elevations is so slow that flow in the channel may be considered to be steady flow, at any instant.

Consideration of the method to be used in summarizing the ultimate solution of the problem is advisable before starting the detailed compu-

tations. Assume that the discharge for every possible pair of reservoir elevations is known, having been determined, say, by actual measurements. It is desired to represent the information graphically in a convenient form. Evidently the values of the discharge can be plotted on a plane, at points whose coordinates represent the corresponding water-surface elevations in the two reservoirs. A series of lines drawn

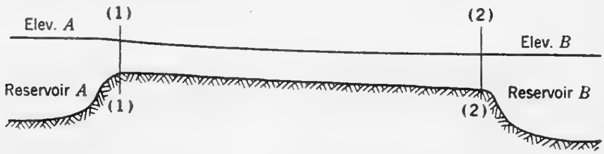


FIG. 803. Channel Connecting Two Reservoirs or Lakes.

through points having the same discharge will facilitate interpolation for intermediate discharges. Figure 804 shows a typical diagram of this type. The water-surface elevations in the two reservoirs are measured from a common datum, and the lowest elevation shown on the diagram is the highest elevation of the bottom of the channel. When the two elevations are the same, there is no flow, so that the line for $Q = 0$ is a straight line extending upward from the origin at an angle of 45° with the axes. Values of Q above this line represent flow from reservoir A to reservoir B; values below the line represent flow in the opposite direction. The diagram will not be symmetrical about the line $Q = 0$ unless the channel is symmetrical about a section midway between the two reservoirs.

The areas shown separated by the dotted lines in Fig. 804 represent different conditions of flow. Throughout areas I and IV the curves for constant Q are straight lines perpendicular to the adjoining coordinate axis. The discharge is independent of the water level in the lower reservoir. This is the result when there is a control somewhere between the two pools. For a channel with steep or adverse slope, the control will be critical flow over the crest, and the water level in the lower pool will not affect the discharge until the control is drowned out. If the channel has a mild slope and is short, areas I and IV will be comparatively small, and may not extend down to the origin, for the critical flow section, which forms at the lower end, can be drowned out by a relatively low water-surface elevation in the lower pool. (If this is possible, a lower elevation should be chosen for the origin of coordinates.) The control section will also be at the lower end for a channel with horizontal bottom. A long channel with a mild slope will have a friction control, the discharge not being affected by the water

level in the lower pool until the *M1* curve forming above the lower pool level reaches the entrance to the channel.

The Q curves in areas I and IV are plotted from computations made as follows: If the control is at the upper end of the channel, the discharge is first computed for various elevations in the upper pool. Profiles for each discharge are then computed to find the highest elevation in the lower pool that will not cause reduction of discharge at the control. If the control is at the lower end of the channel, or at an intermediate point, the discharge is computed for various levels of the water surface just above the control. Profiles are then run upstream to determine the upper pool level. The highest pool level in the lower pool that will not cause reduction of the discharge at the control is finally determined, using profile computations if the control is at an intermediate point in the channel. These computations fix the spacing of the Q curves in areas I and IV, and also the position of the boundaries of these areas.

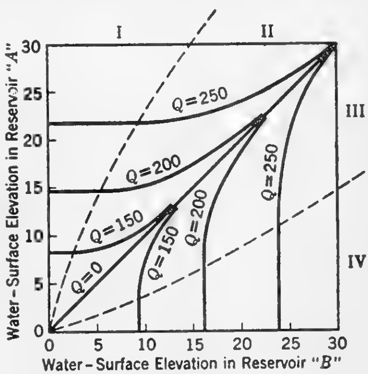


FIG. 804. Discharge in Channel Connecting Two Reservoirs.

In areas II and III the curves for constant discharge are tangent to the Q curves of areas I and IV and asymptotic to the line $Q = 0$. There is no control, and the relative values of the pool elevations have to be determined by profile computations. For uniform channels, the methods described in Chapter VII may be used for the profile computations, with considerable saving of labor, but if the channel is irregular, the step methods of Chapter IX must be used. If the channel is fairly uniform, and in addition is broad and shallow, the methods described in Chapter VI are most advantageous.

PROBLEM

806. A channel 1 mile long and 400 feet wide, with nearly level rocky bottom and vertical riprap sides, connects two lakes, the levels in which may vary up to 10 feet above the channel bottom. Construct a diagram showing the flow in the channel for all possible combinations of levels in the two lakes.

CHAPTER IX

STEP METHODS FOR BACKWATER CURVES

In natural watercourses or artificial channels with frequent changes of section and grade, step methods become more convenient than the integration methods discussed in Chapters VI and VII. The stretch to be studied is divided into short reaches. Trial-and-error computations are made for each reach, based upon the data for the reach and the result of the computation for the preceding reach. This necessitates carrying the computations step by step from one end of the stretch to the other. The reaches need to be short enough to reduce to within permissible limits the error in approximating the true water-surface slope through the reach by the average of the surface slopes at each end, or by the slope corresponding to the average of the hydraulic properties in the reach. Since the radius of curvature of many backwater curves increases continuously in one direction, the error may be systematic, but by using short reaches it can be made much smaller than the error that would be introduced by considering the channel to be uniform.

The amount of departure introduced by lack of uniformity in the channel can be considerable. Figure 901 shows the effect of different degrees of non-uniformity upon what would be a smooth curve in a uniform channel. The integration methods described in Chapters VI and VII (or a special direct step method described in this chapter) can be used for the uniform channel, but for the irregular channels a trial-and-error step method is necessary if the most accurate results are to be obtained.

The step computations for the water-surface profile of Fig. 901(*b*) are started at the dam and carried upstream. For the profile of Fig. 901(*c*), the computations cannot be carried through the reach from end to end, but have to be figured upstream and downstream from the point of control, and upstream from the dam. *Where the depth of flow exceeds the critical, the step computations should be carried upstream, and where the depth of flow is less than the critical, the computations should proceed downstream.* This simple rule serves as a useful guide, but it needs to be supplemented by a knowledge of the methods presented in Chapter VIII if the most complicated cases are to be solved expeditiously.

Fortunately, the majority of problems encountered in practice do not involve complicated water-surface profiles. Often the curve is nothing more than flow at the normal depth, slightly modified by the irregularities of the channel. If the velocity head is comparatively small, the profile can be computed for a short distance in the wrong direction without introducing serious errors. Strict adherence to the rule is desirable, however, for step computations carried in the wrong direction tend inevitably to diverge from the correct elevations.

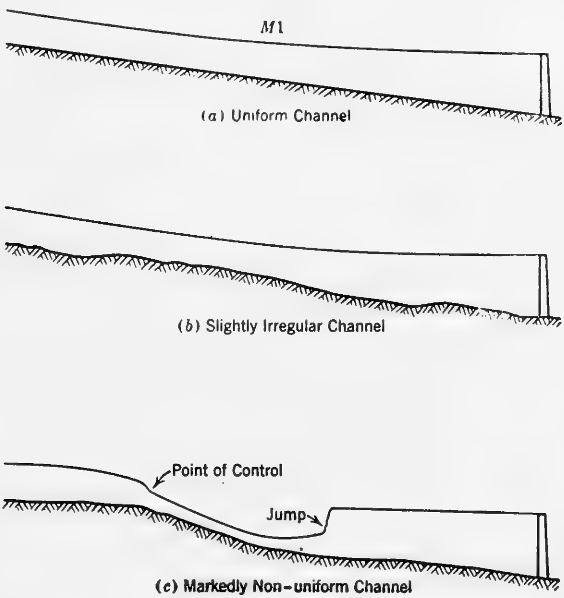


FIG. 901. Effect of Channel Irregularities.

The converse is also true. Computations started at an elevation that is incorrect for the given discharge, and carried in the right direction, will become more nearly correct after every step. This fact is convenient when no elevation is known within or near the reach to be investigated. An elevation may be assumed at a section far enough away from the reach that by the time the step computations have been carried to the initial section of the reach, the elevations will be correct. A check may be had by starting with a different elevation at the distant point. If the distance is sufficient, the computed elevation at the initial section of the reach will be practically the same as before.

The computation of water-surface curves by step methods offers an opportunity for a great variety of procedures. There are but three

methods, however, having significant differences. The choice between them depends upon the type of problem to be solved and the kind of data available. (1) If the data include measurements of the channel shape, a method may be used that has been developed by many authors. It is applicable to the most complicated examples, and always involves trial-and-error computations. This method will be hereinafter referred to as the standard step method. (2) If profiles of the stream at a number of different discharges are available, a convenient method is that of C. I. Grimm, which may be used where velocity head changes are relatively unimportant. Ordinarily trial-and-error computations are used, but they may be eliminated if desired. (3) If the channel is uniform, a direct step method may be used, in which friction and velocity head changes are taken into account. Trial-and-error computations are not necessary.

Any one of the step methods may be used to obtain the basic data for a diagram devised by H. R. Leach which greatly facilitates the computations when a large number of curves for the same discharge, but starting at different elevations, must be obtained. A detailed description of the different methods follows.

Standard step method. The data necessary for the use of the standard step method for computing backwater curves are:

(1) The discharge for which the curve is desired, or data from which the discharge may be determined.
(2) A water-surface elevation at the downstream end of the desired profile, if the depth of flow is greater than the critical; or at the upstream end, if the depth of flow is less than critical.
(3) The cross-sectional area and hydraulic radius at various points along the channel, for all depths of flow within the range expected.
(4) The hydraulic roughness of the various sections of the channel.

If the discharge is not given, it may have to be determined from the head on a spillway, the flow through an obstruction, or other means.

It is not absolutely necessary to have a water-surface elevation at one end of the profile. If none is given, the computations may be started from an assumed elevation above or below the section through which the profile is desired, and the correct elevation found as explained on page 95.

The cross-sectional area and the hydraulic radius at points along the stream may be determined from maps which show contours in the bottom of the channel, from soundings giving the depth of the water, or

from field notes of measurements of cross sections of the channel. In many natural channels there are bays or inlets where the water is quiescent or eddying. If these are included erroneous results will be obtained because their area is not effective in conducting the flow. They should not be included in the values of area and hydraulic radius used in the computations.

The reaches should be short if accuracy is desired, and the sections marking the ends of the reaches should be located at points where the cross-sectional area begins to increase or decrease, where the roughness changes, or where changes of bottom slope occur. It is well to have a man experienced in hydraulic work in charge of the field party; otherwise the length of reach may be dictated by positions where cross-sectioning or sounding is most convenient.

The volume of data necessary to determine the variation of area and hydraulic radius at all of the sections is so great that it becomes important to plan the work of reduction of field notes carefully, and to represent the final results in the form most convenient for the step computations. The obvious method of reduction of the field notes is to plot the data to an undistorted scale, planimeter the areas, and measure the wetted perimeters. It is worth while, however, to introduce certain simplifications developed by J. C. Stevens.[1] The area at low water is planimetered, and the increments of area to be added to obtain the area at successively higher stages are computed from widths scaled off at appropriate elevations. For the wetted perimeter at low stage, an approximate formula is used. Let w represent the horizontal distance between two adjacent soundings, D_1 and D_2 the consecutive depths, and p the corresponding distance along the bottom. If there is no break in the slope of the bottom between the two soundings

$$p^2 - w^2 = (D_2 - D_1)^2 = (\delta D)^2 \qquad [901]$$

Factoring out $p + w$ and solving for p

$$p = w + \frac{(\delta D)^2}{p + w}$$

For the comparatively flat slopes of natural channels, $p + w$ may be replaced by $2w$ without introducing appreciable error.

$$p = w + \frac{(\delta D)^2}{2w} \qquad [902]$$

[1] "Computing Backwater Curves for Surface Slopes in Streams," J. C. Stevens, *Engineering News-Record*, Oct. 1, 1925, v. 95, p. 550.

If the distance between soundings is kept the same

$$P = W + \frac{\Sigma(\delta D)^2}{2w} \qquad [903]$$

Wetted perimeters for stages above low water are obtained by adding increments which are either scaled from the plot of the cross section or computed by slide rule. The computations can be made with an ordinary Mannheim type rule. Set the index of the B scale above the length of the short side of the slope triangle on the D scale. (The short side of the slope triangle will usually be the contour interval.) Set the indicator to the length of the long side of the slope triangle on the D scale. (The long side of the slope triangle will usually be the variable horizontal distance between successive contours.) Opposite the value just set, read on the B scale the ratio of the square of the long side to the square of the short side. Move the indicator over to a value on the B scale greater by 1.00 than the value just read. The length of the hypotenuse of the slope triangle will now appear under the indicator, on the D scale. This is the increment of wetted perimeter for one side of the stream. The increment for the other side is determined similarly. As long as the vertical interval between contours remains the same, the position of the B scale with respect to the D scale does not need to be changed.

After some familiarity with the procedure of computing areas and wetted perimeters has been gained, it will be found possible to dispense with plotted cross sections, the needed values being computed directly from the field notes.

The determination of the hydraulic radius can often be still further simplified. Wetted perimeters are first computed, by one of the methods described above, for two water-surface elevations, high and low. Each wetted perimeter is then divided by the corresponding surface width. If the two coefficients so obtained do not differ appreciably, the wetted perimeters for intermediate stages may be obtained by multiplying the respective surface widths by an interpolated coefficient.

After the areas and wetted perimeters have been computed it is a simple matter to tabulate the corresponding values of water-surface elevation, area, and hydraulic radius for each section.

The roughness of an existing channel is best determined from field data on its area, slope, and discharge. It is rarely possible to obtain these data, and recourse must usually be had to values such as those given in Table 101, and to a study of measurements on similar channels. The coefficients for artificial channels are less uncertain than for natural

ones, but caution should be used to select values corresponding to the conditions which will exist when the flow takes place, for artificial channels may deteriorate rapidly, and in either natural or artificial channels the capacity may vary with the season of the year because of plant growth. The roughness may not be the same throughout the channel, and different values of the coefficient may have to be used in the different reaches, or at different depths of flow in the same reach.

When the choice of coefficient is uncertain, that value should be selected which will give results on the " safe " side. For example, if the investigations are being made to find the minimum draft for navigation, then the value should be used which will give the lowest surface curve. For the common *M1* curve, this will be the smallest possible value of the roughness coefficient. If, on the other hand, possible damages from flooding are being investigated, a large value of the roughness coefficient, at the other end of the range of uncertainty, should be used.

The best way of preparing the elevation, area, hydraulic radius, and roughness data for use in the actual step computations depends upon whether the roughness varies with the elevation, and upon the number of backwater curves (for different discharges or starting elevations) to be obtained. If the roughness does not change with the elevation, and only one or two curves are to be computed, the area-elevation data and hydraulic radius data for each section are plotted to an undistorted scale, and the step computations started without further preliminaries. If the Manning formula is to be used, the step computations may be arranged in tabular form, with column headings as follows:

(1) Section identification, such as " station 35 + 00," " River Street."

(2) Water-surface elevation at the section. A tentative value is entered in this column, to be verified or rejected on the basis of the computations made in the remaining columns of the table. For the first line of the table, this elevation must be known or assumed. When the trial value in the second line has been verified, it becomes the basis for the verification of tentative values in the third line, and so on.

(3) Cross-sectional area corresponding to the water-surface elevation in column (2).

(4) Velocity head corresponding to the given discharge and the area in column (3). (Divide Q by A so that V will be on the D scale opposite the index of the C scale. Then move the indicator to 1.55 on the B scale and read the velocity head on the A scale.)

(5) Elevation of total head line. Obtained by adding the velocity head entered in column (4) to the water-surface elevation of column (2).

(6) Hydraulic radius corresponding to the elevation listed in column (2).

(7) Four-thirds power of the hydraulic radius. This is most conveniently obtained on the ordinary Mannheim type of slide rule. Set R on the CI scale opposite R on the K scale. Read $R^{4/3}$ on the D scale opposite the index of the C scale.

(8) Friction slope corresponding to the area, hydraulic radius, and roughness. By the Manning formula this is

$$S_f = \left(\frac{nV}{1.486}\right)^2 \frac{1}{R^{4/3}}$$

For the assumed condition of constant n,

$$S_f = \frac{V^2}{2g} \times \frac{2gn^2}{2.21} \div R^{4/3} \qquad [904]$$

Thus the friction slope is obtained by multiplying the value of the velocity head in column (4) by a constant, and dividing by the value of $R^{4/3}$ from column (7).

(9) Average friction slope through the reach. This is approximated by taking the arithmetic mean of the value of the friction slope just computed in column (8), and that for the previous step.

(10) Length of the reach. This is the distance between the successive sections, measured along the center line of the stream.

(11) Friction loss in the reach. This is the product of the values in columns (9) and (10).

(12) Eddy loss. No rational method of evaluating the eddy loss is known. It is zero when the velocity head increases through the reach, and some proportion of the difference of velocity heads at the end sections when the velocity head decreases through the reach. Since the amount of the eddy loss is a matter of individual judgment and is roughly approximate at best, the part of the decrease of velocity head to be used can be selected mentally. It should be relatively small where conditions favor a smooth, gradual reduction of velocity.

(13) Elevation of the total head line (a check value). This is computed by adding[2] the values of friction and eddy loss in columns (11) and (12) to the elevation at the lower end of the reach, which will be found in column (13) of the previous step. If the value so obtained does not agree closely with that entered in column (5), the entire line just computed should be neatly crossed out and a new line started with

[2] If the step computations are proceeding in the downstream direction, these losses must be *subtracted.*

a different tentative water-surface elevation in column (2). The unsuccessful trial should not be discarded, for it will serve as a guide in re-estimating the unknown elevation and will provide a partial check, by comparison, on the subsequent computations.

If the value obtained does agree with that entered in column (5), and there are no errors in the computation, the trial water-surface elevation assumed in column (2) was correct, and a value for the water-surface elevation at the next section can be assumed and tested in the next line of computations.

Fort Loudoun Dam – First Construction Stage Cofferdam

Backwater for $Q = 160{,}000$ c.f.s.
Manning Formula with $n = 0.035$

$$\frac{2g \times \overline{0.035}^2}{1.486^2} = 0.0357, \therefore \qquad S_f = 0.0357 \frac{V^2}{2g} \div R^{4/3}$$

1	2	3	4	5	6	7	8	9	10	11	12	13
Station	Water Surface Elevation	Area	Velocity Head	Elevation of Total Head Line	Hydraulic Radius R	$R^{4/3}$	Friction Slope S_f	Average S_f thru Reach	Length of Reach	Friction Loss	Eddy Loss	Elevation of Total Head Line
0+00	766.0	19,700	1.03	767.03	30.7	96.1	.00038					767.03
10+80	765.0	11,500	3.01	768.01	30.6	95.6	.00112	.00075	1080	0.81	0.17	768.01
bridge	765.0	*9,600	4.31	769.31	14.4	35.0	.0044	–	0	0.00	1.30	769.31
~~11+80~~	~~766.8~~	~~12,100~~	~~2.72~~	~~769.52~~	~~31.8~~	~~101.0~~	~~.00097~~	~~.0027~~	~~100~~	~~0.27~~	~~0.00~~	~~769.58~~
11+80	766.9	12,150	2.70	769.60	31.8	101.0	.00095	0027	100	0.27	0.00	769.58
19+50	769.3	22,100	0.82	770.12	21.7	60.4	.00048	.00071	770	0.55	0.00	770.13
~~32+80~~	~~770.5~~	~~31,200~~	~~0.41~~	~~770.91~~	~~25.3~~	~~74.3~~	~~.00020~~	~~.00034~~	~~1330~~	~~0.45~~	~~0.00~~	~~770.58~~
32+80	770.2	30,700	0.42	770.62	25.0	73.0	.00021	.00034	1330	0.46	0.00	770.59
~~54+50~~	~~770.5~~	~~30,500~~	~~0.43~~	~~770.93~~	~~31.0~~	~~97.5~~	~~.00016~~	~~.00018~~	~~2170~~	~~0.40~~	~~0.00~~	~~770.99~~
54+50	770.6	30,600	0.42	771.02	31.1	97.8	.00015	.00018	2170	0.39	0.00	770.98

*Effective area through bridge at 10+80 assumed to be 90% of actual area at constriction.

FIG. 902. Typical Backwater Computation, Standard Step Method.

Based on simple area-elevation and hydraulic radius curves. If curves like those shown in Fig. 903 have been prepared columns (3), (6), and (7) of this table may be omitted. Multiplying the net area under the bridge by 0.9 is equivalent to using $K = 0.9$ in D'Aubuisson's formula, equation (1012).

An example of backwater curve computation by the method just described is shown in Fig. 902. When the Kutter formula is to be used instead of the Manning formula, column (7) should be omitted and a column added for the average velocity, following the one for the area.

If the roughness varies with the water-surface elevation, if there is considerable overbank flow which must be considered separately, or if several backwater curves are to be obtained, a variant of the method just described is more convenient. According to the Manning formula, the friction slope is given by

$$S_f = \frac{n^2Q^2}{2.21A^2R^{4/3}} \qquad [905]$$

Except for the discharge Q, the variables on the right side of equation

(905) are all functions of the elevation of the water surface at the section. Hence a plot may be prepared showing the variation of the friction slope with the water-surface elevation at the section, for the given discharge. Figure 903(a) shows such a plot, for the data from two identical sections of the example given in Fig. 902. After the curve for one value of Q has been computed, others may be obtained by increasing or reducing the abscissa values in proportion to the square of the discharge. A logarithmic abscissa scale is used, so that this is easily

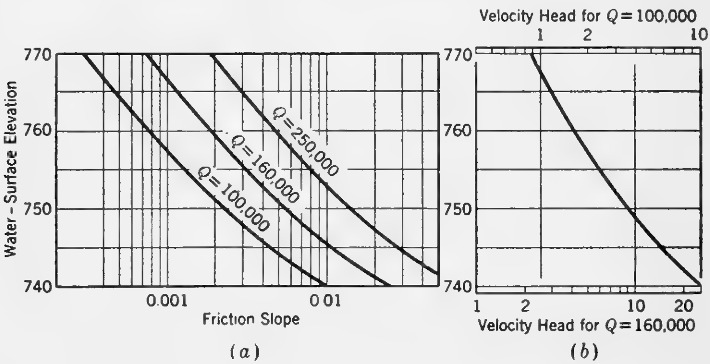


FIG. 903. Friction Slope and Velocity Curves for Stations 10 + 80 and 11 + 80 of the Example Given in Fig. 902.

accomplished by shifting the curve horizontally without changing its shape. An even simpler expedient is to shift the origin of the abscissas. The different scales should be clearly labeled.

Similar diagrams may be prepared for each section to show the variation of velocity head. For a given water-surface elevation, the velocity head also varies with the square of the discharge, so that logarithmic plotting is again convenient. Figure 903(b) illustrates the expedient of shifting the origin of abscissas.

In preparing the diagrams summarizing the hydraulic elements, the range of values computed should be kept within reasonable limits. Unless the curves are needlessly extended beyond the expected range, the time spent in their preparation will be returned with interest during the step computations.

After the velocity head and friction slope data have been summarized in the form shown in Fig. 903, the step computations can be made in essentially the same way as before. Columns (3), (6), and (7) are omitted from the table.

A number of other variants of the standard step method are possible.

All in which both velocity head changes and friction are taken into account will give about the same results. If the velocity of flow is low, so that the velocity head is small compared with the average depth, the computations may be simplified by ignoring the effect of velocity head changes. It is impossible to fix a definite limit for this; proportionately greater velocity head changes may be ignored when the velocity is decreasing in the downstream direction than when it is increasing in the downstream direction. If there is doubt, a check computation should be made in which the velocity head changes are taken into account.

Grimm's method. Grimm's step method[3] may be used when profiles of the stream in its natural state, without the backwater effect, are available. The profiles should be for several different discharges, with water-surface elevations completely covering the range in which the backwater curve is expected to lie. It is thus necessary to have profile data for a larger discharge than that for which the backwater curve is desired. Such data may be synthesized, if not otherwise available, by using one of the methods for extending rating curves described by Hoyt and Grover.[4]

The data for Grimm's method are often cheaper to obtain than the data required for the standard step method. It is most useful when the backwater effect is intermittent, as that caused by navigation dams, or by tributary flow. Where occasional backwater effects are important, continuous records of the profile are kept. The data are always up-to-date, and frequent roughness determinations and cross-section measurements of shifting channels are not necessary, as they might be if the standard step method were to be used.

With respect to the changes of velocity head due to irregularities in the channel, so troublesome in the standard step method, Grimm's method is probably quite accurate. However, it does not take into account the effect of the change of velocity head due to the backwater effect, and for this reason should only be used for those backwater curves in which the velocity is well below critical, and is decreasing in the downstream direction.

In the application of Grimm's method, the stretch is broken up into reaches, which, as for all step methods, should be short. They may be interpolated between the stations where profile observations were taken. Curves showing the relation between discharge and elevation are drawn for each section. Using the subscript n to refer to normal flow con-

[3] "Backwater Slopes Above Dams," by C. I. Grimm, *Engineering News-Record*, v. 100, p. 902, June 7, 1928.

[4] *River Discharge*, by J. C. Hoyt and N. C. Grover, John Wiley & Sons, third edition, 1914.

ditions without the backwater effect, we may write

$$Q_n = \frac{1.486}{n} A R_n^{2/3} S_n^{1/2}$$

according to Manning's formula. For flow with backwater

$$Q = \frac{1.486}{n} A R^{2/3} S_w^{1/2}$$

in which S_w represents the water-surface slope corresponding to the given discharge Q, if velocity head changes due to the backwater are neglected.

From the discharge-elevation curves, determine the value of Q_n corresponding to the known elevation of the backwater curve at the first section. The value of Q for the backwater curve is less, and since the values of A, R, and n are the same, the slope of the backwater curve must be

$$S_w = S_n \left(\frac{Q}{Q_n}\right)^2 \qquad [906]$$

The slope at the first section having thus been determined, the elevation of the backwater curve at the second section may be estimated. This elevation is likely to be in error, for the average slope across the reach is poorly approximated by the slope at one end. The elevation may be used, however, to determine a value of Q_n at the second section, and thus a slope at the second section, after which the elevation may be recomputed, using for the average slope across the reach the better approximation of the mean of the slopes at the first and second sections. Further corrections are seldom necessary. After a little experience, it is found to be possible to avoid revisions by arbitrarily altering the first estimated elevation in the right direction. When the elevation at the second section has been checked. the process is repeated on through the length of the backwater curve.

In making the computations it is more convenient to work with the fall in a given reach than with the slope.

$$F = F_n \left(\frac{Q}{Q_n}\right)^2 \qquad [907]$$

The value of the normal fall F_n is read directly from the profiles.

Illustrative Example

Grimm's method should only be applied to natural watercourses, but to save reproducing numerous curves, and to afford a comparison, the problem of the

example on page 77 will be used to illustrate the method. The channel has a bottom width of 100 feet and side slopes of 1 to 1. Manning's n is 0.022. The uniform grade is 0.0004, and the discharge of 6,220 c.f.s. normally flows at a depth of 10 feet. Determine the backwater curve above a dam at which the depth is 25 feet.

There is no need to draw the flood profiles, for they will be straight lines parallel to the bottom. The discharge-elevation curves will all be similar,

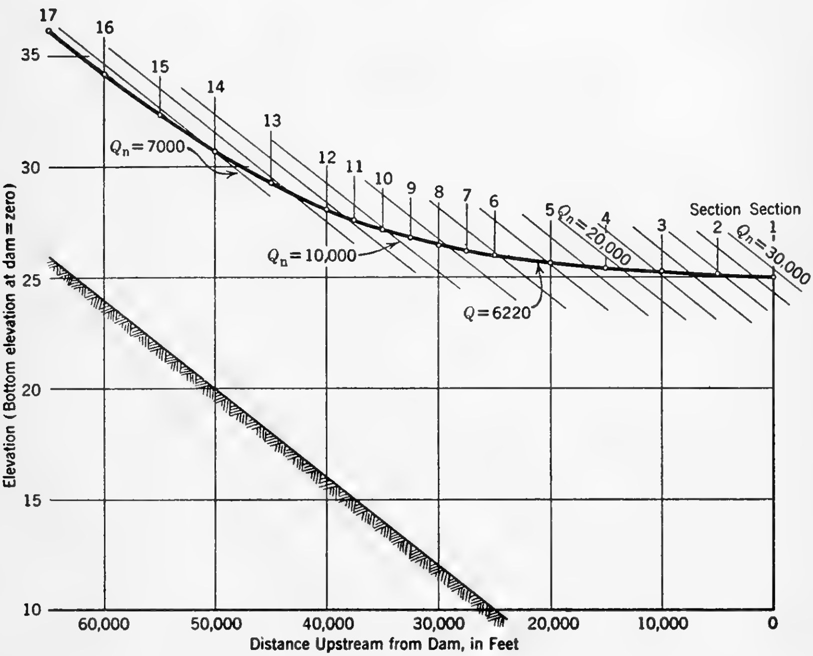


FIG. 904. An *M1* Curve by Grimm's Method.
(See computations in Table 901.)

differing only in elevation. Let the zero datum for elevations be at the base of the dam.

To illustrate the method for natural watercourses, reaches of different lengths will be used. The first reach is taken as 5,000 feet. From Fig. 904 it is seen that Q_n for a normal surface elevation of 25.00 at section 1 is 29,300. The normal fall in reach is 2 feet. The fall corresponding to the slope at section 1 is

$$F = F_n \times \left(\frac{Q}{Q_n}\right)^2 = 2 \times \left(\frac{6,220}{29,300}\right)^2 = 0.090$$

The approximate elevation of the backwater curve at section 2 will be

$$25.00 + 0.09 = 25.09$$

TABLE 901

COMPUTATIONS FOR EXAMPLE ILLUSTRATING GRIMM'S METHOD

Section No.	Distance from Dam	Elevation of Water Surface and Average Fall	Q_n for Water-Surface Elevation	F_n for Q_n	Fall in Reach $F = \left(\frac{Q}{Q_n}\right)^2 F_n$
1	0	25.00	29,300		
		0.104		2.00 2.00	0.090 0.118
2	5,000	25.10	25,600		
		0.138		2.00 2.00	0.118 0.158
3	10,000	25.24	22,200		
		0.186		2.00 2.00	0.158 0.215
4	15,000	25.43	19,000		
		0.255		2.00 2.00	0.215 0.295
5	20,000	25.69	16,200		
		0.350		2.00 2.00	0.295 0.406
6	25,000	26.04	13,800		
		0.221		1.00 1.00	0.203 0.240
7	27,500	26.26	12,700		
		0.263		1.00 1.00	0.240 0.287
8	30,000	26.52	11,600		
		0.312		1.00 1.00	0.287 0.338
9	32,500	26.83	10,700		
		0.367		1.00 1.00	0.338 0.396
10	35,000	27.20	9,900		
		0.425		1.00 1.00	0.396 0.455
11	37,500	27.63	9,220		
		0.489		1.00 1.00	0.455 0.522
12	40,000	28.12	8,610		
		1.182		2.00 2.00	1.044 1.320
13	45,000	29.30	7,650		
		1.445		2.00 2.00	1.320 1.57
14	50,000	30.74	7,020		
		1.66		2.00 2.00	1.57 1.75
15	55,000	32.40	6,650		
		1.815		2.00 2.00	1.75 1.86
16	60,000	34.21	6,450		
		1.895		2.00 2.00	1.86 1.93
17	65,000	36.11	6,340		

The correct elevation will be a little higher, try 25.10. For this elevation at section 2, Q_n is 25,600, and the corresponding fall in reach 1–2 is

$$2 \times \left(\frac{6{,}220}{25{,}600}\right)^2 = 0.118$$

The average fall in the reach, that is, the fall indicated by the average of the falls based upon the computed slopes at each end, is

$$\frac{0.090 + 0.118}{2} = 0.104$$

hence the elevation of 25.10 is satisfactory.[5] The computations are best arranged in tabular form. Table 901 appears to be unnecessarily repetitious, but this is due to the uniformity of the channel used in the example. For a natural channel, repeating values would occur only by accident. They are included in the table for the sake of completeness, in order to show the proper arrangement for irregular channels.

Leach's diagram. The trial-and-error step methods for computing backwater curves in irregular channels are awkward and laborious at best. Yet no simpler methods are known that will give results of comparable accuracy. When studies are being made to determine the economical height of a dam or in other problems where the initial elevation is indeterminate, a large number of backwater curves may have to be computed for the same discharge. In this case Leach's diagram[6] may be used to advantage. The diagram may be prepared after a few curves have been computed by the step method. From it the reach elevations for backwater curves between those used in preparing the diagram may be read directly. Figure 905 shows the construction of the diagram. Figure 905(a) shows the portion of the complete diagram that applies to the first reach. Abscissas are elevations at the lower end of the reach, and ordinates are elevations at the upper end of the reach. After sufficient step computations have been made (at least three) the curve of Fig. 905(a) may be plotted. It shows the elevation at the upper end of the reach for any elevation at the lower end of the reach, within the range of the values plotted.

The curves for a number of reaches may be combined upon a single diagram, as shown in Fig. 905(b). It is most convenient to alternately reverse the coordinates, so that the entire curve may be traced by

[5] The trial-and-error computations may be avoided by the use of nomographs described by I. H. Steinberg, "The Nomograph as an Aid in Computing Backwater Curves," *Civil Engineering*, v. 9, p. 365, June, 1939.

[6] "New Methods for the Solution of Backwater Problems," by H. R. Leach, *Engineering News-Record*, v. 82, p. 768, April 17, 1919.

reading elevations first from one axis, and then from the other, as shown by the dotted line.

Step method for uniform channels. If the channel is uniform, a simplified step method may be used. It is more convenient than the trial-and-error standard step method, but not so convenient as the

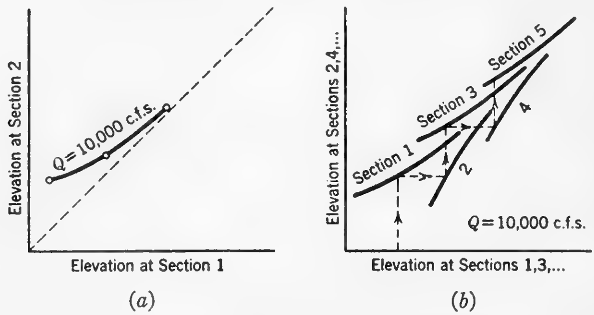


FIG. 905. Leach's Diagram for Different Backwater Curves at the Same Discharge. (*a*) For a single reach. (*b*) For several reaches.

methods described in Chapters VI and VII. It is easily remembered, however, and can be used when tables are not available. For a step of length Δx, Bernoulli's equation may be written

$$S_0 \Delta x + D_1 + \frac{V_1^2}{2g} = S_f \Delta x + D_2 + \frac{V_2^2}{2g}$$

Solving for Δx,

$$\Delta x = \frac{D_2 - D_1 + \dfrac{V_2^2}{2g} - \dfrac{V_1^2}{2g}}{S_0 - S_f} \qquad [908]$$

To use this equation, assume a value of D_2 slightly greater than the known value of D_1. The difference should be small to keep systematic errors to a minimum. The velocity head at each end of the step will be known, as will the bottom slope S_0. The value of S_f may be approximated by the mean of its values at each end of the step. The right-hand member of equation (908) can then be computed, giving the length of the step. Before the computations are started, curves similar to those of Fig. 903 should be prepared. Only one set of curves will be needed for a uniform channel.

Care is needed in the application of equation (908), to insure that positive and negative quantities are correctly interpreted. The computations progress upstream, if the depth of flow is greater than the critical; or downstream, if it is less than the critical. The friction slope S_f may be evaluated by either the Manning or Kutter formulas.

Backwater curves past islands. It is often necessary to compute a backwater curve for a stream which is divided into two channels by an island. The division of flow between the two channels is unknown. To solve this problem, first estimate the division of flow. Then compute backwater curves past each side of the island to the point where the streams unite again, one set of curves with a smaller proportion of the flow going on one side of the island than was estimated, and the other set with a greater proportion. The elevations of the water surface at

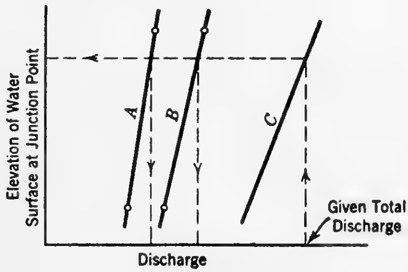


FIG. 906.

the junction point are plotted against discharge, as shown on Fig. 906, and lines (A and B) drawn representing the relation for the two sides. A third line C is drawn adding the abscissas of the lines A and B, thus giving the relation between total discharge and elevation at the junction point. The ordinate on this line opposite the point whose abscissa is the total discharge of the stream is the required elevation of the junction of the two channels. If extrapolation is necessary, or if extreme precision is desired, it may be advisable to compute a third set of backwater curves, using the proportions indicated.

This method of computing the flow and the backwater curves past islands is applicable only when there is tranquil flow throughout. If the velocities at the point where the flow divides are above the critical, the proportion of flow going to each side of the island will be dependent upon flow conditions upstream from the point of division.

CHAPTER X

BENDS, TRANSITIONS, AND OBSTRUCTIONS

The theories of the backwater curves and of the hydraulic jump, as elaborated in the preceding chapters, will suffice for most practical problems in determining the longitudinal water-surface profiles of canals and rivers. Cases will occasionally arise, however, in which water-surface curves are needed that cannot be computed under the assumptions of either of these theories. Though not even the simplest hydraulics of all the possible problems of this type has been worked out on a mathematical basis, certain of them have been studied enough to yield results of practical value. It is the purpose of the present chapter to discuss the more important of these in some detail. A good knowledge of the preceding chapters, and especially of the first four chapters, is a necessary prerequisite to a complete understanding of the material presented in this chapter.

Flow around bends: velocity less than the critical. One of the troublesome but unavoidable characteristics of open channels is the presence of curves in alignment. When water flows in a straight uniform channel the transverse profile is probably a horizontal line. Various observers have claimed that the water in the middle of the stream is higher than at the banks. If waves are present, they are much larger in the middle than near the banks, and their crests are therefore correspondingly higher; but it has not yet been satisfactorily proved that the average surface elevation is any greater at the center.

On a bend or curve the transverse surface profile cannot be level. Water, like all matter, when in motion moves in accordance with Newton's first law of motion, that is, it moves in a straight line unless deflected by the action of some unbalanced force. If water moves in a curve, there must be an unbalanced force acting against the water and directed towards the center of curvature. Let Fig. 1001 represent the transverse cross section of a stream at a bend. Consider that portion of the stream whose cross section is represented by the area $ABFE$. As this portion flows around the bend it is deflected toward the center of curvature on the inside of the bend. This deflection is caused by the excess of pressure on the face BF over the pressure on the face AE. The excess of pressure can exist only when the water surface at B is

slightly higher than at A. The amount of excess can be calculated by the formula for centrifugal force when the velocity of the moving water and the curvature of its path are known. For an actual stream such calculations are subject to some degree of uncertainty because the velocity varies throughout the cross section, and because the radius of curvature cannot be precisely determined. Subject to these uncertainties, however, the calculated difference in surface elevation between the inner and outer banks at a bend agrees with the observed difference within the limit of possible error in the observation. The surface at the outer bank, for flow at velocities well below the critical, is rarely higher than the surface at the inner bank by more than a few inches.

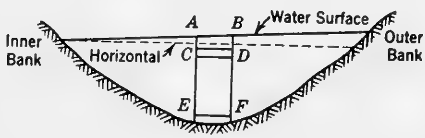


FIG. 1001. Channel Cross Section Illustrating Conditions in Bend of River.

Formulas for the difference in water-surface elevation between the inner and outer banks of a stream flowing around a curve at less than the critical velocity can be derived as follows: Consider the forces acting on an element of the stream represented by $ABEF$, Fig. 1001. Assume that all parts of this element have the same velocity and are moving in circular paths around the same center of curvature.

Let b = breadth of the stream,
r = radius of curvature of flow at the element $ABFE$,
r_1 = radius of curvature of inner bank,
r_2 = radius of curvature of outer bank,
dr = distance AB,
y = depth of element AE,
dy = height of B above A,
V = velocity of water,
w = weight of unit volume of water,
R = radius of curvature at center of stream.

The centrifugal force acting on the element $ABFE$ is equal to the excess pressure on the face BF over the pressure on the face AE, due to the height of the surface at B above the surface at A. Considering the element to have a length of unity up and down stream, its volume is ydr, its weight is $wydr$, and its mass is $(w/g)ydr$. The excess pressure on the face BF is $wydy$. From mechanics it is known that centrifugal

force $= wV^2/gr$. Therefore,

$$\frac{wyV^2dr}{gr} = wydy$$

or

$$dy = \frac{V^2dr}{gr} \qquad [1001]$$

To integrate this equation the value of the velocity at all points across the river must be stated in terms of r. Formulas may be obtained under a variety of assumptions.

A fairly good approximation is obtained by assuming V constant at the average velocity, and assuming r to be constant at the value for the center of the stream. Then the

$$\text{Difference in elevation of the two banks} = \frac{V^2b}{gR} \qquad [1002]$$

This will always give too small a value because the effect of the filaments with the higher velocities more than offsets the effect of the slower filaments, since the velocity enters to the second power in the integral.

If the actual velocity distribution across the stream is known, the width may be divided into several sections, the difference in surface elevation computed for each section using its appropriate velocity and radius, and the total difference found as the sum of the differences for the separate sections.

If the velocity is zero at each bank and has a maximum value V_m at the center, varying in between according to a parabolic curve:

Total difference in surface elevation =

$$\frac{{V_m}^2}{g}\left[\frac{20}{3}\frac{R}{b} - 16\frac{R^3}{b^3} + \left(\frac{4R^2}{b^2} - 1\right)^2 \log_e \frac{2R+b}{2R-b}\right] \qquad [1003]$$

The effect of the variable velocity distribution throughout a cross section is, probably, to increase slightly the actual radius of curvature followed by the moving water in going around a bend, especially for a short curve. Thus, for a curve of 90 degrees the effective radius for the whole stream is probably as great as the radius of the outer bank; but for a curve of 180 degrees the effective or actual radius cannot much exceed the average of the radii of the inner and outer banks.

According to F. C. Scobey[1] the lower elevation of the water at the

[1] "The Flow of Water in Flumes," by Fred C. Scobey, U. S. Department of Agriculture *Technical Bulletin* 393.

inside of a bend cannot be taken advantage of in laying out the walls of flumes, for the water surface there is more wavy than at the outside of the bend, and more freeboard is needed.

Another detail regarding the curved motion around a bend should be noted at this place. Referring again to Fig. 1001, the excess of pressure required at D over that at C varies as the square of the velocity of the moving water, other things being equal. But the layer near the bottom has a much slower velocity than a layer near the top like CD. It follows then that the same superelevation at B cannot furnish precisely the required excess pressure for both layers CD and EF. What really happens is that the two layers do not bend around the same center of curvature and do not have the same radius of curvature. The superelevation at B takes an intermediate value, and the motion of each layer adjusts itself to the corresponding pressure difference. The layer EF has a shorter and the layer CD a greater radius of curvature than the average. As a result the fastest moving water gradually shifts toward the outer bank as it moves around the bend, and there is a compensating creep of the slower moving water near the bed of the channel towards the inner bank. This effect cumulates with the length of the bend, so that in a long bend, such as a semicircular curve, finally all the water with highest velocity is near the outer bank while in the inner half of the cross section the velocity of the water is much slower. This phenomenon was observed and studied in a small experimental channel by Professor James Thomson about 1870, and has become known as the " spiral flow " in bends.

The spiral or helicoidal flow set up in the bend is clockwise if the stream curves to the left, looking downstream, or counterclockwise, if the stream curves to the right. Helicoidal flow was observed and studied by Blue, Herbert, and Lancefield[2] in a sharp bend in the Iowa River. The spiral-forming tendency of the sharp bend or turn was found to be so pronounced that it reversed the direction of helicoidal flow due to a milder bend in the opposite direction which was immediately upstream. The cross section of the Iowa River at these bends is comparatively deep and narrow. Where the cross section is wide and shallow the spiral is much attenuated, and may be so weak as to be difficult, if not impossible, to observe.

Consideration of the helicoidal flow throws interesting light upon the question of the energy loss due to bends. If the bend is followed by a long tangent, the helicoidal flow will persist for some distance downstream, until it finally dies out because of the effect of friction. It is

[2] " Flow Around a River Bend Investigated," by Blue, Herbert, and Lancefield, *Civil Engineering*, v. 4, p. 258, May, 1934.

evident that the entire energy of the spiral flow will be dissipated, so that a greater friction loss would be expected than in an equal length of straight channel without a bend. Another reason for increased energy loss in bends is that the high velocity part of the stream comes closer to the banks than in normal straight channel flow. The actual dissipation of energy due to both of these effects takes place largely in the straight reach below the bend, where the helicoidal motion and turbulence die out. If the bend is followed by another bend of equal radius and length, but in the opposite direction, and the spiral set up by the first bend is neutralized in the second, the energy loss might be less than for two equal bends in the same direction. However, tests of pipe bends flowing full, in which two spirals form, show exactly the opposite effect, with greater loss for two adjacent bends in the opposite direction than for two equal adjacent bends in the same direction. Whatever the result is in any given case, it is evident that in determining the energy loss due to curvature, the spacing and sequence of the bends must be considered, as well as their radius and central angle. On the basis of his tests of the Tiger Creek Flume, which has considerable curvature alternating in direction, Scobey[3] suggests that the value of Kutter's n be increased 0.001 for each 20 degrees of curvature in 100 feet of flume. Until more data are available, the effect of spacing and sequence of bends, and of their radii, cannot be evaluated, and Scobey's suggestion can only be safely followed with flumes similar to the Tiger Creek Flume.

The next point for consideration is the condition at the junction points of straight channels and bends. In actual rivers there is always some sort of a gradual transition curve at both the beginning and ending of a curve, but it will be simpler to discuss a theoretical channel in which two tangents are connected by a circular curve of constant radius. Since the transverse profile is level on the approaching tangent but must be inclined around the bend, the question arises as to how the change takes place from one state to the other. Perhaps, theoretically, the level cross section could be transformed to the slanting shape most simply by raising the water surface at the outer bank and lowering it at the inner bank by equal amounts. Blue, Herbert, and Lancefield found an eddy at the outside of the sharp Iowa River bend, indicating a rise in the water surface at the beginning of the curve. This is rarely found except in the sharpest bends. Usually the longitudinal slope at the outer bank flattens or becomes practically level for a short distance. The remainder of the required transverse slope results from a rather sudden depression in the water surface at the inner bank at the beginning of the curve.

[3] *Loc. cit.*

An important secondary effect of the changes in surface elevation at the beginning of a curve is the disturbance of the velocity distribution in the cross section. The relative surface elevation at the outer bank is accompanied by a reduction in velocity in that region, while the surface depression at the inner bank produces a local acceleration of velocity. This condition may usually be observed at the beginning of a sharp bend in a river channel, and produces an appearance as though the rapidly moving water had moved over from the center of the channel to a course near the inner bank. It should be remembered, however, that this appearance is partly deceptive. The rapidly moving water near the inner bank is in a large part the same water which was near the bank farther up stream, moving slowly, and which has been accelerated at the approach of the curve.

At the lower end of the curve another adjustment of the transverse surface profile must be effected in joining to the straight channel. As before, it seems to be easier for the higher surface, now at the outer bank, to be depressed, than for the lower surface, now at the inner bank, to be raised, in order that the two may be adjusted to a common level. Observation of actual channels shows that the longitudinal slope often becomes nearly level at the inner bank for a short distance, and the major part of the necessary surface adjustment is obtained by a rather sudden drop of the water surface along the outer bank. Just as was true at the beginning of the curve, the sudden drop of the water surface is necessarily accompanied by a corresponding acceleration of velocity, but with this important difference. At the upper end of the curve, the water receiving the acceleration was in that part of the stream near the bank which had previously possessed a relatively low velocity; therefore, even after being accelerated its velocity was not conspicuously high. At the lower end of the curve, however — particularly on a long curve of 90 degrees or more — the water receiving the sudden acceleration near the outer bank is the water already possessing the highest velocity of any in the cross section, having been thrown outward towards the outer bank by centrifugal action during its progress around the curve; therefore, after being accelerated, its velocity is conspicuously high, often greater than at any other point within a considerable distance either up or down stream. This high velocity close to the outer bank often produces heavy erosion. Though erosion often takes place near the inner bank at the beginning of the curve, it is generally much less extensive than that to be seen near the outer bank at the lower end of a curve.

Curves in open channels are objectionable, first, because they tend to increase frictional losses; and second, because they increase the danger of serious local erosion. Where bank protection becomes neces-

sary, it is most needed on the outer bank in the lower part of the curve, and to a less degree on the inner bank at the beginning of the curve.

It frequently happens that navigation improvements such as training walls or dams partially obstruct the main stream in such a way that the current is deflected away from a re-entrant angle or pocket. The water surface in the re-entrant corner will have a higher elevation than that of the main stream. The pocket will contain a large eddy. If the stream lines for a given condition of flow are known or can be approximately located, integration based on equation (1001) can be applied to compute roughly the amount of superelevation needed to deflect the main stream as it curves out to miss the re-entrant angle.

Flow around bends: velocity greater than the critical. Flow around curves in alignment, when the velocity is greater than the critical, is entirely unlike that which occurs when the velocity is less than the critical. The water surface, instead of being smooth and showing only a slight superelevation, is likely to have a broken and varying transverse profile, with greater difference in elevation at the inner and outer banks than would be indicated by any of the formulas used to estimate the superelevation in low-velocity flow. High-velocity flow around bends is uncommon, and is, therefore, likely to be unfamiliar even to those who have had much experience with bends having low-velocity flow. For this reason many of the flume bends for high-velocity flow that have been constructed in the past do not operate satisfactorily. Study of the problem during the last ten years, however, has shown the reason for the difficulties which have been encountered.[4] Methods are being developed for the design of bends having greatly improved flow conditions, and also for the correction of bad flow conditions in existing bends.

Consider first a circular curve of constant radius between two long tangents in a rectangular channel having a bottom which is level transversely. The high-velocity flow approaching the curve is level transversely. If the side of the channel were suddenly moved inward a small distance, a wave would start which in quiet water of the same depth would travel across the channel, with a velocity equal to the critical, and be reflected back and forth until it finally died out. With respect to the moving water, the side of the channel is being continuously " pushed in " starting at a point A, Fig. 1002. This causes a wave to travel across the channel, which at the same time is swept downstream by the swift current. The wave begins along the line AM, and the

[4] " Curvilinear Flow of Liquids with Free Surface at Velocities above that of Wave Propagation," R. T. Knapp and A. T. Ippen, *Proceedings of the Fifth International Congress for Applied Mechanics*, p. 531. John Wiley and Sons, 1938.

water surface up to that line is undisturbed. The angle β_0 that the line makes with the tangent extended beyond point A is approximately: arcsin $(\sqrt{gD}/V)$. It is known as the " wave angle."

Continuing past point A, the side of the channel is being " pushed in " against the flow at an increasingly rapid rate, so that the water surface rises higher and higher around the outside wall until another effect, which we shall consider next, sets in. Water-surface contours in the area AMD will be a series of lines approximately parallel with the wave front AM.

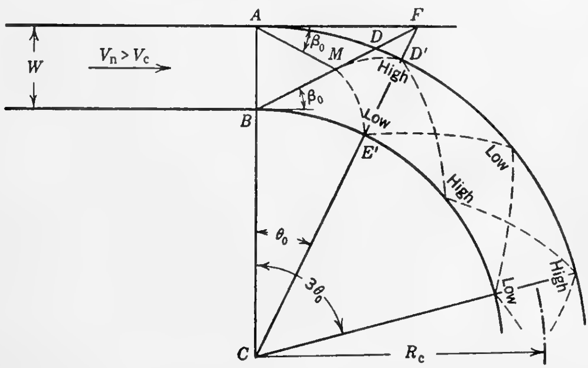


FIG. 1002. Plan View of High-Velocity Flow Around a Bend in an Open Channel.

At point B a negative wave is started which travels across the current along the line BM. The water line along the inside wall drops lower, and the water-surface contours in the area immediately downstream tend to parallel the line BM.

The two waves do not stop when they meet near the center of the channel. It is a familiar characteristic of waves that they may cross, be reflected from a wall, and recross, without much loss of form or even change of velocity (in quiet water). Thus the negative wave travels on across the surface of the positive wave, diminishing its height, and the positive wave travels downward across the negative wave, filling in the depression. If the water had the same depth and velocity that it has in the area ABM the negative wave would begin to strike the outside wall at point D. The increased depth tends to make the wave front curve toward the left as it approaches point D, while the changed direction of the velocity makes the wave front curve towards the right. The net result, as observed by Ippen and Knapp in their experiments, is that the wave front curves slightly to the right, first striking the outside wall at point D'. The central angle corresponding to the arc AD' is

best approximated by the angle θ_0, or ACF. From this point downstream the influence of the negative wave which started at point B begins to affect the height of the water surface, which had been rising along the outer wall from A to D', so that at point D' the water-surface profile along the outside wall will have a maximum. At some point farther downstream the greatest intensity of the negative wave will reach the outside wall and the water-surface profile will have a minimum.

In a similar manner the positive wave which started at A will reach the inside wall at point E', and cause the profile along the inside wall, which drops continuously from B to E', to turn upward.

The positive and negative waves continue to be reflected back and forth across the channel, causing the profiles along both inner and outer walls to have a series of maximums and minimums at angles θ_0, $3\theta_0$, $5\theta_0$, from the beginning of the curve. Moreover, it is likely that the effect will continue past the end of the curve into the tangent below, giving rise to the possibility of aggravated conditions at the next curve downstream.

The phenomenon is much like that which would occur if a railway car with undamped springs ran suddenly onto a circular curve without superelevation or spiral. The equilibrium position of the car body would be changed instantaneously from horizontal to a slant corresponding to the speed of the car and the radius of the curve. The car body, however, could not assume the required slant instantly. By the time it did reach the equilibrium position it would have considerable angular momentum about its longitudinal axis, which would carry it well beyond the equilibrium position before the springs could start it back, thus giving rise to a series of oscillations. At the end of the curve, the equilibrium position would suddenly change back to the horizontal. This would start another series of oscillations, which might, if the curve were of just the right length, cancel or diminish the series started at the beginning of the curve, but which would be just as likely, in general, to reinforce partially or even double the effect of the first series.

Ippen and Knapp give equations for determining the phase length of the disturbance and the depth of flow along the outside wall up to the first maximum. These may be summarized as follows. (The notation has been changed to conform to the authors', except as shown on Fig. 1002.) The central angle to the first maximum is given by

$$\theta_0 = \arctan \frac{W}{(R_c + W/2)\tan\beta_0} \qquad [1004]$$

This follows from Fig. 1002, by geometry. The depths along the walls

at an angle $\theta < \theta_0$ are given by

$$D = \frac{V_n^2}{g} \sin^2 (\beta_0 \pm \theta/2) \qquad [1005]$$

the positive sign giving depths along the outside wall and the negative sign depths along the inside wall. The depth at the maximum height of the first disturbance is obtained by substituting the value of θ_0 given by equation (1004) for θ in equation (1005).

Knapp and Ippen also describe measures which they found to be effective in preventing the formation of excessively high standing waves in bends of channels having a rectangular cross section.[5] For the correction of undesirable flow conditions in existing channels, diagonal sills may be installed at the beginning and end of the curve. For new designs good flow conditions may be insured by the use of circular transition curves, or by banking.

The purpose of the diagonal sills, and also of the circular transition curves, is to introduce a counter-disturbance of just the right magnitude, phase, and shape to neutralize the undesirable oscillations which would form at the changes of curvature. It was found that this could be accomplished by building simple rectangular sills diagonally across the bottom of the channel. The details of the design had to be determined experimentally.

Circular transition curves of comparatively simple design were also found to function satisfactorily. The transition curve should have twice the radius of the central curve. It should curve in the same direction and be a half wave length long. The central angle of the transition curve is given with sufficient accuracy by the formula

$$\theta_t = \tan^{-1} \frac{W}{R_t \tan \beta_0} \qquad [1006]$$

in which $R_t (= 2R_c)$ is the radius of the transition curve. If the radius and length of the transition curves have these values the counter-disturbance set up will be of the correct magnitude and phase relation to minimize the height of the standing waves in the curve. A similar circular transition at the lower end of the curve is necessary to prevent disturbances in the downstream tangent.

The method of banking permits equilibrium conditions to be set up without the introduction of counter-disturbances. It was the most effective method tested. For equilibrium, the cross-slope should be

[5] "Experimental Investigation of Flow in Curved Channels — Abstract of Results and Recommendations" (2 volumes), A. T. Ippen and R. T. Knapp. July 6, 1938. U. S. Engineer Office, Los Angeles.

that theoretically required by the radius and average velocity, but it must be introduced gradually. If it is increased linearly in a distance L along the channel, a spiral wall transition should be used. The required cross-slope should be obtained by depressing the bottom along the inside wall rather than raising it along the outside wall. For this reason the method is not as practical as the use of circular transition curves. The minimum length of the transition should be

$$L_t = 15WS_c \tag{1007}$$

where S_c is the cross-slope

$$S_c = \frac{V^2}{Rg}$$

The transition spiral should be constructed according to the equation

$$RL = R_c L_t = \text{a constant} \tag{1008}$$

The corrections are to be made for flow at maximum design capacity. The flow pattern at lower flows might conceivably be unsatisfactory. Ippen and Knapp state that the compound or transition curve method is the best in this respect, with the flow pattern remaining remarkably constant at all stages.

Changes in cross section. The variable section connecting one uniform channel to another of different cross-sectional form is called a transition. Let us first consider the simplest form of transition, that of a change of width of a rectangular channel. The change is assumed to be effected smoothly, and in a short distance, so that there is no additional contraction of the jet, beyond that guided by the walls, and no appreciable friction loss. The bottom is level throughout.

Let W_1 = width of channel upstream from the contraction or enlargement,
W_2 = width of channel downstream from the contraction or enlargement,
D_1 and V_1 = upstream depth and velocity,
D_2 and V_2 = downstream depth and velocity.

A change in the width of the channel will result in a change in the ratio of velocity head to depth, so that unless W_1 and W_2 are equal $\frac{V_1^2/2g}{D_1}$ and $\frac{V_2^2/2g}{D_2}$ will be unequal. If there is no loss of energy, as assumed,

$$D_1 + \frac{V_1^2}{2g} = D_2 + \frac{V_2^2}{2g}$$

After dividing through by D_2 and rearranging terms, we obtain

$$\frac{V_2^2/2g}{D_2} = \frac{D_1}{D_2}\left(1 + \frac{V_1^2/2g}{D_1}\right) - 1 \qquad [1009]$$

But since Q is constant,

$$V_1D_1W_1 = V_2D_2W_2$$

and

$$\frac{D_1}{D_2} = \frac{W_2V_2}{W_1V_1} = \left(\frac{W_2}{W_1}\right)^{2/3}\left(\frac{\dfrac{V_2^2/2g}{D_2}}{\dfrac{V_1^2/2g}{D_1}}\right)^{1/3}$$

Substituting in equation (1009),

$$\frac{V_2^2/2g}{D_2} = \left(\frac{W_2}{W_1}\right)^{2/3}\left(\frac{\dfrac{V_2^2/2g}{D_2}}{\dfrac{V_1^2/2g}{D_1}}\right)^{1/3}\left(1 + \frac{V_1^2/2g}{D_1}\right) - 1 \qquad [1010]$$

Solution of equation (1010) for numerical computation is awkward. The authors have plotted the equation in Fig. 1003, which shows the relationship between $\dfrac{V_1^2/2g}{D_1}$, $\dfrac{V_2^2/2g}{D_2}$, and W_2/W_1 over the range of practical values of the variables. Examination of Fig. 1003 shows that for every value of $\dfrac{V_1^2/2g}{D_1}$ there are two possible values of $\dfrac{V_2^2/2g}{D_2}$, except when $\dfrac{V_2^2/2g}{D_2}$ is equal to one-half — that is, when the flow in the downstream section is critical. This is to be expected, for unless the flow in the downstream section is critical, two alternate depths of flow are possible. The diagram also shows that for any given ratio of velocity head to depth in the downstream section, there are two possible values of the ratio for the upstream section, unless the flow in the upstream section is critical.

Since there is no loss of energy, flow could be in either direction, and the diagram should be symmetrical about the line $\dfrac{V_1^2/2g}{D_1} = \dfrac{V_2^2/2g}{D_2}$, with the values of the W_2/W_1 curves on one side of the line equal to the exact reciprocals of the curves symmetrically opposite, on the other side of the line. The curves shown in the diagram are not exactly symmetrical, however, for the values of W_2/W_1 plotted, above and below unity, are not reciprocals of each other.

To illustrate further the use of Fig. 1003, let us consider a numerical example. Suppose that a rectangular channel 10 feet wide narrows down smoothly, in a short distance, to a width of 9 feet. If the depth

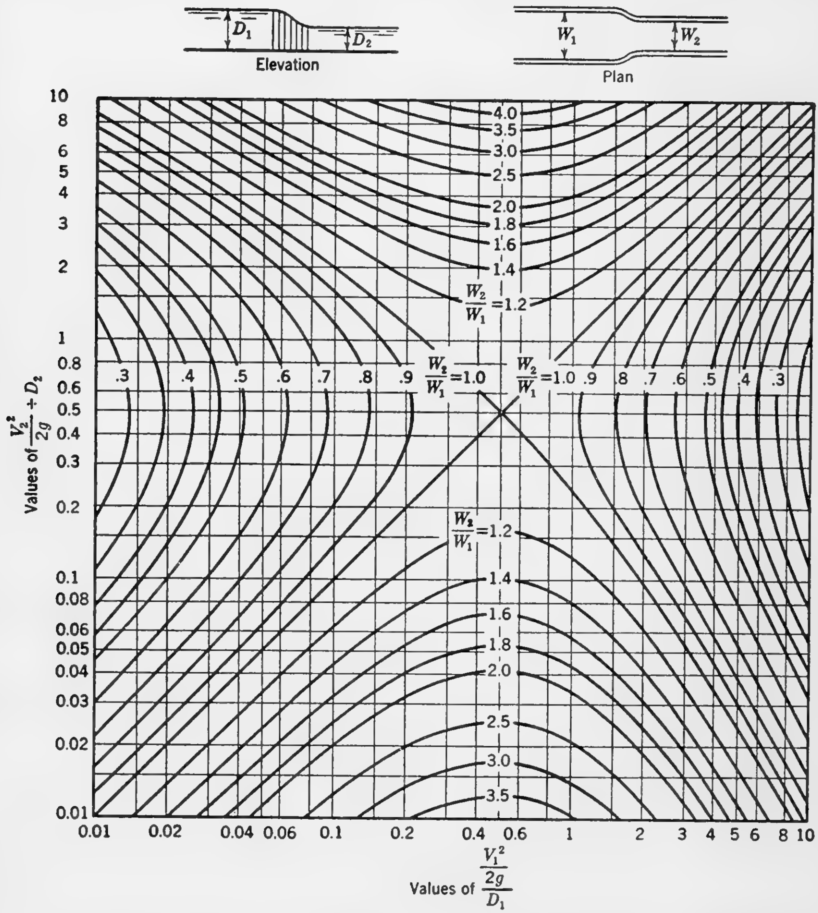


Fig. 1003. Ratios of Velocity Head to Depth Above and Below a Change of Width in a Rectangular Channel.

in the 10-foot section is 6 feet, and the velocity 5 feet per second, what is the depth in the section of 9-foot width? To solve this problem with the aid of the diagram, first compute the following:

$$\frac{V_1^2/2g}{D_1} = \frac{0.389}{6} = 0.0649, \quad \frac{V_1^2}{2g} + D_1 = 0.389 + 6.0 = 6.389, \quad \text{and}$$

$$\frac{W_2}{W_1} = \frac{9}{10} = 0.9$$

Entering the diagram of Fig. 1003 with $\dfrac{V_1^2/2g}{D_1} = 0.0649$ and $W_2/W_1 = 0.9$, we find that

$$\frac{V_2^2/2g}{D_2} = 0.085 \text{ or } 2.25$$

The total head at the downstream section is assumed to be the same as that of the upstream section, and the depth of flow may therefore be either

$$D_2 = \frac{6.389}{1.085} = 5.83$$

or

$$\frac{6.389}{3.25} = 1.97$$

A check by computing the total head based on the velocities corresponding to these depths shows that they are accurate to within less than one per cent.

Choice of the alternate depth which applies in any given situation depends upon flow conditions in the transition and in the channels upstream and downstream from the transition. Analysis of flow conditions in the channels may be accomplished by the methods explained in Chapter VIII. This should be done before starting analysis of flow conditions in the transition itself. For example, it would be impossible for a transition to convert flow at greater than the critical depth to flow at less than the critical depth if the downstream channel was incapable of carrying the high-velocity flow away from the transition.

Results of the analysis of flow conditions in the channels may indicate that only one depth of flow in each channel is possible, or they may indicate more than one possibility. In the former case the transition may or may not cause local disturbances depending upon the details of its design. In the latter, a detailed analysis of flow conditions in the transition may show which alternative flow profile will exist. At times, where a small difference in the total head seriously affects the possibilities, the result is uncertain.

In analyzing the flow conditions in the transition we recognize the following possibilities:

(*DD*) Flow deeper than critical throughout.

(*DDn*) channel narrows in the downstream direction.

(*DDw*) channel widens in the downstream direction.

(*SSn&w*) Flow shallower than critical throughout.

(*DSn&w*) Transition from deeper than critical to shallower than critical.

(*SDn&w*) Transition from shallower than critical to deeper than critical.

Not all of the possible cases are equally reliable in operation. Detailed comments follow.

Operation of case *DDn* is predictable unless the downstream depth is near the critical, when the flow may cross the critical in the transition and come out below at the stage which is alternate to that desired. For example, consider where $\dfrac{V_1^2/2g}{D_1} = 0.06$ and $W_2/W_1 = 0.6$. From Fig. 1003 it is seen that $\dfrac{V_2^2/2g}{D_2}$ should equal 0.35, but if the external flow conditions permitted it, a value of 0.75 would be likely. Theoretically, the value of 0.75 would not be possible unless the transition narrowed down to 0.59 and then widened to 0.60, but actually the small deviation of the depth necessary to cross the critical would occur easily. If W_2/W_1 equalled 0.8, $\dfrac{V_2^2/2g}{D_2}$ would be 0.11, with the flow still much deeper than the critical, so that the alternate value 1.9 would be very unlikely. When the depth approaches the critical in a *DDn* transition, there is always danger of the flow changing to *DSn*. This may occur even when the downstream flow must be at greater than critical depth to fit the tailwater, with a hydraulic jump occurring in or below the transition.

In the operation of case *DDw*, only one flow condition is possible. The energy loss may be a large proportion of the difference of the two velocity heads if the angle of expansion is too great.

Case *SSn* is predictable if the angle of contraction is less than the wave angle, and if the depth does not approach the critical too closely. If it does, the depth below the transition may rise above the critical, starting a condition of flow (*SDn*) which is very unstable, and which may give way to a hydraulic jump.

Case *SSw* is stable. The angle of divergence of the walls should be less than the wave angle, or transverse waves will be started which will continue to disturb the water surface for some distance below the transition.

Case *DSn* forms occasionally when not expected, as noted in the discussion of case *DDn*. This form can be relied on when called for by the general flow conditions, provided the downstream depth is only slightly less than the critical. If the downstream depth is to be appreciably

less than the critical, the transition must have a "throat," that is, it must narrow down to the value of W_2/W_1 corresponding to the critical depth at an intermediate value of $\frac{V_2^2/2g}{D_2}$ vertically above the value of $\frac{V_1^2/2g}{D_1}$ on Fig. 1003. The transition then becomes similar to that shown in Fig. 203, with flow from right to left.

Cases *SDn* and *w* seldom occur, though they are theoretically possible. Figure 203, with flow from left to right, could illustrate either case, depending upon where the transition ended at the right. Hinds illustrates a similar transition in which the bottom is humped.[6] It is not known whether a successful transition of this type has ever been built. What usually happens is that a hydraulic jump forms upstream from the transition, and flow through the transition becomes *DDn* or *w*.

In general, the transition profiles for which the water surface is level or falling, going downstream, are stable and dependable. Those leading to a rising or uphill water surface are likely to be undependable. The flow conforms to the theoretical best when the water-surface profile through the transition, computed by applying Bernoulli's equation to a succession of short reaches, is a smooth curve tangent to the normal flow at each end, and when the maximum angle of divergence or convergence of the walls is less than the wave angle.[7]

Bridge piers and pile trestles as channel obstructions. Bridges and pile trestles, even when built without approach embankments, obstruct part of the area of flow of the stream. If the normal depth of flow of the stream is deeper than the critical, the water upstream from the bridge is deeper than the normal, while the depth a short distance downstream differs but little from the normal. The resulting drop in the water surface is known as the "backwater" caused by the bridge. It causes a backwater curve which extends on upstream, and which may, if it is high enough and deep enough, result in flood damage in excess of that which would be caused by normal flow in the stream.

If the normal depth of flow of the stream is less than the critical, a condition which does not often occur where backwater is of impor-

[6] "Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," Julian Hinds, *Engineering News-Record*, Nov. 25, 1920, p. 1034.

[7] "The Hydraulic Design of Flume and Siphon Transitions," Julian Hinds, *Trans. Am. Soc. Civil Engr.*, v. 92 (1928), p. 1423. Many practical suggestions for the design of transitions are given in this paper. According to Hinds, friction losses in well-designed transitions where the flow is accelerated may be taken as 5 per cent of the difference of the velocity heads, and where it is decelerated, as 10 per cent of the difference of the velocity heads.

tance, the piers " split " the high-velocity flow, throwing water and spray into the air. Though the flow is much disturbed downstream, there is no upstream effect, and in the center of the spans the flow continues under the bridge at normal depth until it encounters the waves started by the piers. Except when the bridge and its piers so constrict the flow that the water immediately upstream from the bridge is dammed up to greater than critical depth, it is impossible for the bridge piers to affect the upstream water surface.[8] If the flow is constricted to this extent, the water surface upstream from the bridge will follow an *S1* curve for a short distance, starting at a hydraulic jump with its downstream end where the depth of the *S1* curve becomes sequent to the normal depth of the stream.

The more common condition of low-velocity flow obstructed by bridge piers or pile trestles has been the subject of several experimental investigations. For an extensive bibliography, as well as the description of a large number of tests of different kinds of piers and trestles, see " Bridge Piers as Channel Obstructions,"[9] and " Pile Trestles as Channel Obstructions,"[10] both by D. L. Yarnell. The experiments seem to show the need of further subdivision of low-velocity flow. The most logical basis is that proposed by Yarnell:

Class A — Flow in contracted section is at greater than critical depth (tranquil flow).

Class B — Flow in contracted section is at less than critical depth (shooting flow).

The Rehbock classification, proposed by T. Rehbock when he found that the results of some 2,000 small-scale tests could not be formulated in a single category, is as follows:

Class 1 — Ordinary or " steady " flow, in which the water passes the obstruction with very slight turbulence or no turbulence.

Class 2 — Intermediate flow, in which the water passing the obstruction displays a moderate degree of turbulence.

Class 3 — " Changed " flow, in which the water passing the obstruction becomes " completely " turbulent.

The boundaries of the Rehbock classification are defined by empirical equations, while the equation of the boundary of the Yarnell classi-

[8] " Backwater Wave in the Spillway Channel, Gatun, Panama Canal," Edward C. Sherman, *Engineering News*, v. 67, p. 813, April 25, 1912.

[9] *Technical Bulletin* 442, U. S. Department of Agriculture.

[10] *Technical Bulletin* 429, U. S. Department of Agriculture.

fication may be easily derived on the basis of the definition of the classes. Both are plotted in Fig. 1004. Values on the boundary between the Yarnell classifications may be checked by reference to Fig. 1003.

Yarnell's tests, however, indicate (as he himself points out) that the Rehbock classification is more useful than his classification, though there is no conclusive evidence that a single formula might not be developed which would have satisfactory accuracy over the entire range of tranquil normal flow in the unobstructed section.

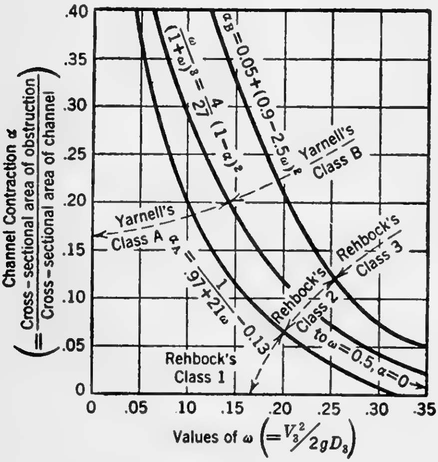


FIG. 1004. Classifications of Flow in Channels Obstructed by Bridge Piers.

(After *U.S.D.A. Tech. Bull. 442.*)

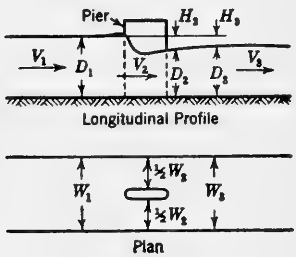


FIG. 1005. Diagram of Bridge Pier.

Yarnell tested the types of piers most commonly used in American practice with contractions of 11.7, 23.3, 35, and 50 per cent, and pile trestles with contractions of 12.3 to 16.2 per cent.

His tests showed that for flow in Class 1 and Class 2, the Nagler formula showed the least variation with the amount of contraction, over the range tested.[11] Figure 1005 shows the notation used in the Nagler formula, which is

$$Q = KW_2\sqrt{2g}\left[D_3 - \frac{\theta V_3{}^2}{2g}\right]\sqrt{H_3 + \frac{\beta V_1{}^2}{2g}} \qquad [1011]$$

The coefficient θ is a correction coefficient, the factor $\theta V_3{}^2/2g$ being intended to correct D_3 to give a smaller depth of flow similar to that at

[11] " Obstruction of Bridge Piers to the Flow of Water," F. A. Nagler, *Trans. Am. Soc. Civil Engr.*, v. 82 (1918), p. 334. See also " Experiments on the Flow of Water through Contractions in an Open Channel," E. W. Lane, *Trans. Am. Soc. Civil Engr.*, v. 83 (1919), p. 1149.

the most contracted section. In all of Yarnell's comparisons, the value of θ was taken as 0.3. (It is likely that θ should vary, approaching zero as the contraction percentage approaches zero, and assuming higher values if the flow is in Class 3.) The coefficient β, which corrects

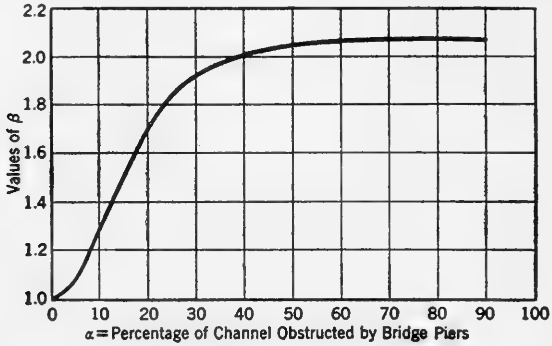


FIG. 1006. Values of β in Nagler Bridge-Pier Formula.

for the velocity of approach, is considered to vary as shown in Fig. 1006, which is taken from Nagler's paper.

Values of K in the Nagler formula, for Class 1 and Class 2 flow between various types of bridge piers with different percentages of contraction, are shown in Fig. 1007. The values shown are for common

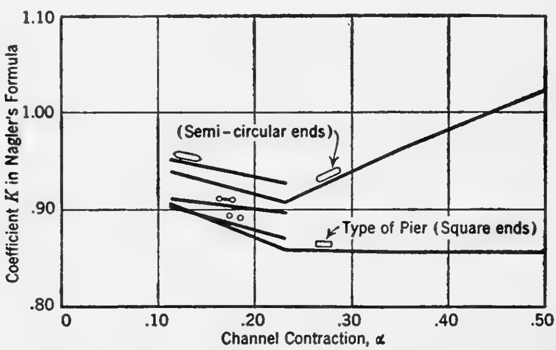


FIG. 1007. Yarnell's Data on Variation of K in Nagler Bridge-Pier Formula.

Average values for Class 1 and Class 2 flow.

types of piers, with length equal to four times the width, set parallel with the current. Tests on longer piers showed that lengths up to thirteen times the width gave values for K less by only 2 per cent. Setting standard piers at an angle of 20 degrees with the current de-

creased K only 5 per cent (net width of pier, not including its projection, subtracted in computing W_2). All these values are from Yarnell's tests, and though they lack verification in the field, they are believed to be the best available.

For Class 3 flow, the D'Aubuisson formula fits Yarnell's experimental data better than the Nagler formula. D'Aubuisson assumed that the water-surface drop at the contracted section is merely the difference in the velocity heads, and he did not distinguish between the depths D_2 and D_3. In practical cases there will be little difference. Using the notation of Fig. 1005, the formula may be written

$$Q = KW_2D_3\sqrt{2gH_3 + V_1^2} \quad [1012]$$

Yarnell gives the following values of K in D'Aubuisson's bridge pier formula, for Class 3 flow between standard bridge piers.

Square noses and tails	0.95
Semicircular noses and tails	
Parallel with current	1.03
10-degree angle	0.93
20-degree angle	0.88
90-degree triangular noses and tails	0.98
Twin-cylinder piers, with or without diaphragms	0.99
Lens-shaped noses and tails	0.95

The value of K decreased from 3 to 5 per cent for longer piers, with lengths up to thirteen times the width. The addition of batter to the ends of the piers slightly increased their hydraulic efficiency, that is, raised the value of K.

The following coefficients, based on Yarnell's tests, are recommended for use in computing the backwater due to pile trestles:

	Nagler K for Class 1 and Class 2 Flow	D'Aubuisson K for Class 3 Flow
Single-track 5-pile trestle bent		
in line with current	0.90	0.96
10-degree angle	0.90	...
20-degree angle	0.89	...
30-degree angle	0.87	...
Double-track 10-pile trestle bent	0.82	0.88
Two single-track 5-pile trestle bents	0.79	0.86

The amount of channel contraction is to be taken as the average diameter of the piles plus the thickness of the sway bracing. When the bents are at an angle with the current, the channel contraction is to be taken as the same as for the same bent placed parallel with the current. The effect of the angle is included in the coefficient.

Yarnell computed Nagler, D'Aubuisson, and Rehbock coefficients for his tests, which covered all three classes of flow. The Nagler coefficient showed the least dispersion in Classes 1 and 2, and the D'Aubuisson coefficient the least dispersion in Class 3. Some variation with velocity was observed, and since the highest velocities reached were only about 6 feet per second, the need for further tests, especially field measurements, is indicated. It should be noted that Yarnell's tests did not include percentages of contraction of less than 11.7 per cent. For smaller percentages the D'Aubuisson formula should be used.

In computing the backwater due to a bridge or trestle, the formulas must be applied to the average properties of the whole cross section, and not to individual sections, no matter how much they differ, for the height of the backwater is the same at all parts of the cross section.

Illustrative Examples

1. A stream discharging 50,000 cubic feet per second, at flood stage, flows under a bridge having 5 rectangular piers 12 by 50 feet which reduce the normal width of waterway from 400 feet to 340 feet. If the average normal depth of flow in the uncontracted section is 20 feet, what will be the height of the backwater caused by the bridge?

First summarize the known factors, and compute those needed to classify the flow.

$$Q = 50{,}000, \quad W_1 = 400, \quad W_2 = 340, \quad \alpha = \frac{60}{400} = 0.15, \quad D_3 = 20,$$

$$V_3 = \frac{50{,}000}{20 \times 400} = 6.25, \quad \frac{V_3^2}{2g} = 0.605, \quad \text{and} \quad \omega = \frac{\dfrac{V_3^2}{2g}}{D_3} = \frac{0.605}{20} = 0.0302.$$

From Fig. 1004 the flow is seen to be in Class 1, so that the Nagler formula should be used.

$$Q = KW_2\sqrt{2g}\left[D_3 - \frac{\theta V_3^2}{2g}\right]\sqrt{H_3 + \frac{\beta V_1^2}{2g}} \qquad [1011]$$

The value of K, from Fig. 1007, is taken as 0.88, and β, from Fig. 1006, as 1.5. Using $\theta = 0.3$, substituting the known factors, and solving,

$$H_3 + 1.5\,\frac{V_1^2}{2g} = 1.02$$

The value of D_1 is not known, so that V_1 is not known. However, $V_1^2/2g$ cannot be much less than $V_3^2/2g$, so let $V_1^2/2g = 0.60$ for a first approximation. Then

$$H_3 = 0.12 \text{ feet}$$

from which $D_1 = 20.12$, $V_1 = 6.21$, and $V_1^2/2g = 0.60$, so that the value of $H_3 = 0.12$ feet satisfies the formula.

2. Measurements were obtained on a flash flood which flowed under a single-track 5-pile trestle bridge 200 feet long. The percentage of obstruction was 14.5 per cent, and the bents were nearly in line with the current. The average depth of the water downstream from the trestle, which was not clogged with debris, was 4 feet. The observed fall through the trestle was 0.7 feet. What was the discharge?

First summarize the known factors. $W_2 = 200 - (200 \times 0.145) = 171$, $\alpha = 0.145$, $D_3 = 4$, $H_3 = 0.7$, and $D_1 = 4.7$. Substituting these values into the D'Aubuisson formula, equation (1012), and using $K = 0.96$, we obtain

$$Q = 656\sqrt{43.6 + V_1^2}$$

Assuming $V_1^2 = 40$, a first trial $Q = 656\sqrt{83.6} = 6{,}000$, from which

$$V_1 = \frac{Q}{A_1} = \frac{6{,}000}{200 \times 4.7} = 6.38, \quad \text{and} \quad V_1^2 = 40.7$$

Assuming $V_1^2 = 41$, second trial $Q = 656\sqrt{84.6} = 6{,}030$, from which $V_1^2 = 41.2$, by the same method. A third trial with $V_1^2 = 41.4$ gives $Q = 6{,}050$, $V_1 = 6.44$, and $V_1^2 = 41.4$, which checks. The class of flow should now be computed to see if use of the D'Aubuisson formula is justified.

$$V_3 = \frac{6{,}050}{200 \times 4.0} = 7.56, \quad \frac{V_3^2}{2g} = 0.89, \quad \text{and} \quad \omega = \frac{0.89}{4} = 0.22$$

Entering Fig. 1004 with $\omega = 0.22$, and $\alpha = 0.145$, the flow is seen to be in the Class 2 area, not far from the Class 3 boundary. Yarnell's data indicate that the Nagler formula is more reliable in this area. Substituting the given data into the Nagler formula, equation (1011), and using $K = 0.90$

$$Q = 1{,}233\left[4.00 - 0.3\frac{V_3^2}{2g}\right]\sqrt{0.70 + 1.47\frac{V_1^2}{2g}}$$

Using for $V_3^2/2g$ and $V_1^2/2g$ the final values obtained in the D'Aubuisson solution, $Q = 5{,}910$. This is enough less than 6,050 to demand another approximation. Values of $V_3^2/2g$ and $V_1^2/2g$ corresponding to $Q = 5{,}910$ might be used, but in order to hasten the convergence of the successive trials, try $Q = 5{,}700$. This gives $V_3^2/2g = 0.79$, $V_1^2/2g = 0.57$, and $Q = 5{,}710$, which is close enough. The answers by the two formulas differ by about 6 per cent. The D'Aubuisson formula was used first because it is more convenient. Since the flow is Class 2, the result obtained using the Nagler formula is to be preferred.

PROBLEMS

1001. A rectangular flume 10 feet wide, of smooth concrete, must have a curve with radius of about 200 feet and central angle of 40 degrees at the bottom of a long

grade of 1.5 per cent. The maximum discharge will be about 300 c.f.s. Design the curve. How high should the walls be built?

1002. A bridge which has three sets of twin cylindrical piers 6 feet in diameter crosses a stream which is normally 300 feet wide and 15 feet deep when the discharge is 30,000 c.f.s. What will be the height of the backwater caused by the bridge at this discharge?

CHAPTER XI

SLOWLY VARIED FLOW

Unsteady flow in which the changes with respect to time occur slowly, so that dynamic effects are negligible, is very similar to steady flow. With the exception of the law of continuity, which needs to be revised to take into account any storing up or releasing of water, the same equations that were used for steady flow will apply. Flow of this type is frequently met in engineering practice, most commonly in connection with the so-called " routing problems."

Although there is no definite numerical criterion for the boundary between slowly varied flow and unsteady flow, the status of most problems can be determined by the test described at the end of this chapter. No satisfactory methods for solving problems in general unsteady flow have yet been developed, though certain special cases have been found to be amenable to mathematical solution.[1]

Level-pool routing with invariable stage-discharge relation. The outlet from a reservoir or lake has an artificial or natural control, so that for every possible value of the water-surface elevation in the lake, there is a definite single value of the outflow. The water surface in the lake is assumed to be level, and the area at each possible elevation to be fixed, so that the elevation-storage relation is also single-valued and invariable. The inflow hydrograph is given and it is required to determine the outflow hydrograph.

In passing, it should be noted that the assumption of slowly varied flow is implicit in the assumption of a level pool. If the inflow rate increased so rapidly that a large wave was started at the upper end of the pool, and the wave took an appreciable length of time to travel to the point of outflow, this assumption would not be valid.

It should also be noted that the inflow is not the rate of discharge relative to the (moving) section where the inflowing stream intersects the pool level, but rather the rate of discharge relative to a fixed station. Moreover, all of the water entering the reservoir above the point of outlet must be considered.

[1] " The Hydraulics of Flood Movements in Rivers," Harold A. Thomas, *Engineering Bulletin*, Carnegie Institute of Technology, Pittsburgh, Penna. 1937.

The relationship between the elevation of the water surface and the discharge through the outlet may be represented by an equation of the form

$$o = Bh^n \tag{1101}$$

or of the more general form

$$o = f_1(E) \tag{1102}$$

In these equations o is the outflow, or rate of discharge through the outlet, B is a constant which depends upon the size and hydraulic properties of the outlet, h is the elevation of the water surface above the zero-elevation of the outlet, and n is an exponent, ordinarily either 1/2, for orifice-type outlets, or 3/2, for weir-type outlets. Often the relationship is not expressible by an empirical equation, and has to be tabulated or plotted graphically, in which case the elevation of the water surface E may be measured from any convenient datum. This case is represented by equation (1102). For outlets of simple geometrical shapes, the methods explained in elementary texts on hydraulics will suffice for the determination of the relationship between the water-surface-level and the discharge, but for complicated or natural outlets, one of the following methods may have to be used: (1) field measurements of elevation and discharge, (2) model studies, (3) construction of rating curves by the methods explained in Chapter VIII.

The capacity-depth information for the pool, obtained by computation from survey data giving the areas at regular intervals of elevation, is tabulated or plotted graphically. Vertical distances are preferably measured from the same datum that was used in the outflow-depth relationship. Thus

$$S = f_2(h) \tag{1103}$$

or

$$S = f_3(E) \tag{1104}$$

where S represents the total amount of water stored when the water surface is at elevation E or is at a distance h above the zero elevation of the outlet. It is usually possible to obtain an empirical equation representing this relationship, but this is not necessary for the method which is now to be explained.

The inflow hydrograph is usually specified by giving values of the inflow at regular intervals of time. The hydrograph may be long-time, representing a continuous record of inflow, or it may only include one flood. At any rate the given values follow each other at brief enough intervals that the general shape of the sharpest flood peak is well defined. The outflow hydrograph, which is the result desired, should preferably be obtained in terms of values of the outflow at the same regular inter-

vals of time. Since the inflow is independent of the other variables, the inflow hydrograph may be represented by the equation

$$i = f_4(t) \tag{1105}$$

where i represents the rate of inflow and t represents the time.

We are now reaay to establish the differential equation, a modified form of the law of continuity, the solution to which will be the desired outflow hydrograph. In a brief interval of time a volume of water idt will flow into the reservoir. During the same interval of time a volume of odt will flow out of the reservoir. If idt is greater than odt the difference will be stored in the reservoir, so that we may write

$$idt = odt + dS \tag{1106}$$

in which $i = f_4(t)$, $o = f_1(E)$, and $S = f_3(E)$. If odt is greater than idt, dS will be negative, indicating that the storage is decreasing. We wish to obtain the relationship $o = f_5(t)$. Even though the given functional relationships are defined by simple equations it is impossible to obtain a solution of the differential equation, except in certain special cases. The only one of these having any practical value is when

$$i = f_4(t) = \text{a constant}$$

or, in other words, when the inflow does not vary.[2] If the inflow is allowed to vary, solution of the differential equation is impossible unless the other relationships have simple forms almost never found in practice.

Though this result may seem to preclude any chance of solving the problem at hand it actually points the way towards an easy solution. The computations must be made in steps, each step so short that during its duration the inflow may be considered constant. Since values of the inflow are ordinarily given at brief regular intervals — regular as a matter of convenience, and brief in order to define clearly the shape of flood peaks, a step-by-step solution is most convenient. The first step methods to be used were awkward and laborious, requiring trial-and-error solution of each step, but later methods introduced a direct procedure which greatly reduced the work of computation.[3] The method presented here is the result of further study, pointed toward reducing

[2] "Functional Design of Flood Control Reservoirs," C. J. Posey and Fu-Te I, *Trans. Am. Soc. Civil Engr.*, v. 105 (1940), p. 1638.

[3] "Hydraulics of the Miami Flood Control Project," S. M. Woodward, *Technical Reports* of the Miami Conservancy District, Part VII, Chapter VII; "Flood Routing," Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder, *Trans. Am. Soc. Civil Engr.*, v. 104 (1939), p. 275.

the work of computation to an absolute minimum. The values of inflow are set on an easily made slide rule and the corresponding values of outflow are read directly, with no intermediate computations.

Assume that the inflow i is constant during a step Δt, and that the average outflow o is equal to the mean of the outflow rates o_1 and o_2 at the beginning and end of the step. Then, by comparison with equation (1106)

$$i \cdot \Delta t = \frac{o_1 + o_2}{2} \cdot \Delta t + \Delta S \qquad [1107]$$

This can be written

$$i \cdot \Delta t = \frac{o_1 + o_2}{2} \cdot \Delta t + S_2 - S_1$$

or

$$i \cdot \Delta t + (S_1 - \tfrac{1}{2} o_1 \cdot \Delta t) = (S_2 + \tfrac{1}{2} o_2 \cdot \Delta t) \qquad [1108]$$

If Δt is taken as a constant, then

$i \cdot \Delta t$ is a function of the inflow,

$S_1 - \frac{1}{2} o_1 \cdot \Delta t$ is a function of the water-surface elevation at the beginning of the step, and

$S_2 + \frac{1}{2} o_2 \cdot \Delta t$ is a function of the water-surface elevation at the end of the step.

The water-surface elevation at the beginning of the step is, of course, the same as the water-surface elevation at the end of the previous step. In order to start the routing computations, then, the initial water-surface elevation must be known. As a matter of fact, it is not essential that the initial elevation be known precisely, for errors due to assuming an incorrect value will disappear rapidly as the routing computations proceed.

The procedure of the routing computations is now evident. Knowing, from the preliminary computations, the relations $S = f_3(E)$ and $o = f_1(E)$, and also the elevation at the beginning of the first step, the value of $(S - \frac{1}{2} o \cdot \Delta t)$ at the beginning of that step may be computed. From the given inflow hydrograph, the value of $i \cdot \Delta t$ for the step is computed. Adding this to $(S - \frac{1}{2} o \cdot \Delta t)$ gives the value of $(S + \frac{1}{2} o \cdot \Delta t)$ at the end of the step, from which the value of the elevation at the end of the step can be computed. The process is repeated for each of the succeeding steps. The computations are facilitated by preparing beforehand tables or charts showing the relationships

$$S - \tfrac{1}{2} o \cdot \Delta t = f_6(E)$$

and

$$S + \tfrac{1}{2} o \cdot \Delta t = f_7(E)$$

The greatest convenience of computation, however, is obtained by constructing a special slide rule for solving equation (1108) directly. This can best be described by means of a numerical problem.

Illustrative Example 1101

The area-depth survey data for a large reservoir are shown in colum·· (1) and (2) of Table 1101. Assuming that the reservoir has a fixed orifice-type ·t-let, the discharge through which is given by the equation $o_o = 5{,}650(E - 68$0 ², and an overflow spillway with discharge $o_w = 1{,}520(E - 900)^{3/2}$, route t flood inflow hydrograph shown in columns (8) and (9) of the table through th reservoir, starting with the reservoir empty, and carrying the routing past the maximum stage in the reservoir. Construct a slide rule for step-by-step determination of the stages in the reservoir.

Values of the storage capacity at different elevations are based upon the elevation-area data of the first two columns of the table and are shown in column (3) of the table, converted to c.f.s. days, a unit of volume also abbreviated as

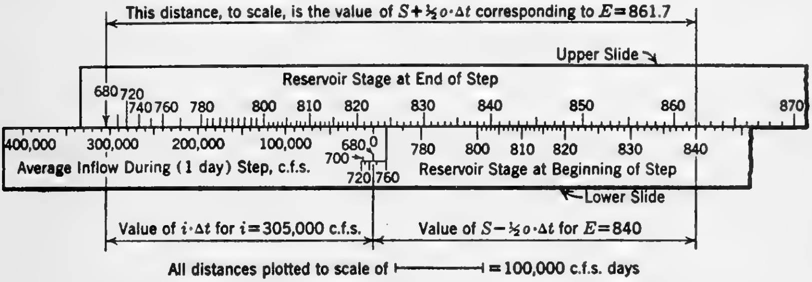


Fig. 1101. Slide Rule for the Reservoir of Illustrative Example 1101.
Graduated in terms of reservoir stage.

d.s.f., and representing the volume of a cubic foot per second flowing for 24 hours. The values of outflow shown in column (4) include not only the discharge through the orifice-type outlet, but also the discharge over the spillway crest for elevations above 900.

The time interval between the inflow values given in column (9) is one day, frequent enough to define the inflow hydrograph and therefore frequent enough to define the outflow hydrograph, which will be less sharply peaked than the inflow hydrograph. The length of step Δt is accordingly chosen equal to 1 day. The computations in columns (5), (6), and (7) then follow the formulas of their headings.

After the values in columns (6) and (7) have been obtained the "slide rule" may be constructed. The divisions are marked in pencil upon two smooth, straight strips of wood,[4] and are labeled as shown in Fig. 1101. The computed

[4] "Parting stop," purchasable at any lumber yard, is cheap and convenient. If the rule is to be preserved or used for some time, a coat of clear linoleum lacquer, applied after it has been graduated, is desirable.

points are plotted to scale, but the intermediate divisions between them are spaced equidistant. The rule as shown is set for the step during which the average inflow is 305,000 c.f.s., for the left end of the upper slide is set opposite 305,000 in the left scale of the lower slide. Opposite 840, the reservoir elevation at the beginning of the step, the elevation at the end of the step, 861.7, is read on the upper slide. It is obvious that the slide rule solves equation 1108 automatically. The complete routing computations made with the aid of the rule are shown in column (10). The maximum stage reached is 902.4. The routing computations have not been carried beyond 16 days. The flood of this example is chosen to be large enough to cause the water to rise above the spillway level, a circumstance which would not be permitted in the design of an effective flood control reservoir.

Notes on construction of flood-routing slide rules. There is a temptation, where the graduations are not uniform, to attempt curvilinear interpolation of the intermediate points. The writers have found this to be inadvisable, for straight-line interpolation of the intermediate points is much easier, and almost invariably gives more accurate results. If greater accuracy is desired than can be obtained with straight-line interpolation, more points should be computed. Essentially, the two scales with non-uniform graduations are made up of the sum (or difference) of two variables, one, the storage, and the other, a constant times the discharge. At elevations where either of these is changing rapidly, computed values should theoretically be more closely spaced. Usually both will change rapidly near the origin, but since even fairly large percentage errors at low stages have inappreciable effect after high stages are reached, it is seldom necessary to compute intermediate values for the low stages. Immediately above a spillway crest, however, the discharge will vary considerably for a small difference in water-surface elevation, and since the stage is high, it is desirable to compute values at close intervals for plotting the scales. In the example shown in Table 1101, two-foot intervals were used above the spillway crest, while 20-foot intervals were used below.

Another effect, besides that just mentioned, must be taken into account if greater accuracy is desired for the lower stages. Referring to Table 1101 and Fig. 1101, it is seen that the " elevation at beginning of step " scale has some negative values corresponding to the lower elevations. This happens when $\frac{1}{2}o \cdot \Delta t$ is larger than S, and it may give rise to inconsistent results in the routing at low stages. The negative values can be eliminated, if it is deemed advisable, by using a smaller value of Δt.

It should seldom, if ever, be necessary to obtain precise results for the low stages in a large reservoir. If it were necessary, the most con-

TABLE 1101

COMPUTATIONS FOR RESERVOIR OF ILLUSTRATIVE EXAMPLE 1101

1	2	3	4	5	6	7	8	9	10
Elev. E	Area	Storage Capacity S	Outflow o	$\frac{1}{2}o \cdot \Delta t$ ($\Delta t = 1$ day)	$S - \frac{1}{2}o \cdot \Delta t$	$S + \frac{1}{2}o \cdot \Delta t$	Time to Middle of Step	Inflow at Middle of Step	Reservoir Stage at End of Step
Feet	*Acres*	*c.f.s. days*	*c.f.s.*	*c.f.s. days*	*d.s.f.*	*d.s.f.*	*Days*	*c.f.s.*	*Feet*
680	0	0	0	0	0	0	0	1,000	681
700	300	1,000	25,200	12,600	−11,600	+13,600	1	20,000	712
720	700	6,000	35,700	17,800	−13,800	+23,800	2	105,000	773
740	1500	17,000	43,700	21,800	−4,800	38,800	3	200,000	811
760	3000	40,000	50,500	25,200	+14,800	65,200	4	265,000	840
780	5200	81,000	56,500	28,200	+52,800	109,200	5	305,000	861.7
800	8200	149,000	61,900	30,900	118,100	179,900	6	315,000	879.1
820	12300	252,000	66,800	33,400	218,600	285,400	7	290,000	891.3
840	17400	402,000	71,400	35,700	366,300	437,700	8	225,000	899.5
860	23400	608,000	75,800	37,900	570,100	645,900	9	150,000	902.4
880	30300	880,000	79,800	39,900	840,100	919,900	10	85,000	902.2
900	38400	1,228,000	83,800	41,900	1,186,100	1,269,900	11	40,000	900.0
902		1,271,000	88,460	44,230	1,226,800	1,315,200	12	10,000	895.8
904		1,314,000	96,720	48,360	1,265,600	1,362,400	13	5,000	891.2
906		1,358,000	107,230	53,615	1,304,400	1,411,600	14	2,000	886.7
908		1,401,000	119,790	59,895	1,341,100	1,460,900	15	1,000	882.0
910	43350	1,444,000	133,800	66,900	1,377,100	1,510,900	16	1,000	876.9

venient method would probably be to construct two rules, one suited to the reservoir as a whole, and the other for the bottom portion only.

The slide rule should be planned so as to minimize the work of computation. If the rule is correctly planned no computations of any kind, other than setting and reading the rule, need to be made in the actual routing. For example, if daily values of inflow are given corresponding to gage readings made each day at a certain time, say noon, averaging

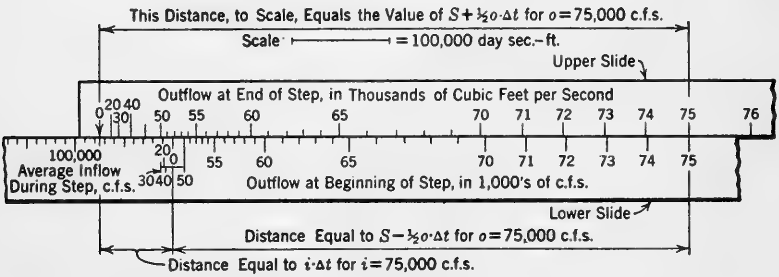


FIG. 1102. Flood-Routing Slide Rule for Example 1101.

Graduated in terms of outflow. The setting shown in this figure illustrates the condition of steady flow, with inflow and outflow equal.

computations may be eliminated by making the duration of a step run from midnight to midnight. The inflow values taken at noon can then represent the average inflow during the step. The outflow values subsequently obtained are for midnight, not noon, a fact which should be remembered if the inflow and outflow hydrographs are to be plotted on the same diagram.

If values of outflow are desired as a result of the routing computations, the labor of converting values of water-surface elevation to outflow may be eliminated by graduating the slide rule in terms of outflow instead of elevation. This can be done, for $(S - \frac{1}{2}o \cdot \Delta t)$ and $(S + \frac{1}{2}o \cdot \Delta t)$ are single-valued functions of the outflow, as well as of the elevation. The computations for a slide rule to solve illustrative problem 1101 in this way are shown in Table 1102, and the rule is shown in Fig. 1102. If needed for some special purpose, a similar rule could be made which would give successive values of the storage.

After the scales have been graduated, a partial check on the plotting, which will disclose certain common types of errors, is easily made. For the rule shown in Fig. 1101, set equal values of the elevation, on the upper and lower scales, opposite each other, and for the rule of Fig. 1102, set equal values of the outflow opposite each other. This represents the condition of steady flow with the inflow equal to the outflow, so that

TABLE 1102

COMPUTATIONS FOR OUTFLOW HYDROGRAPH FOR ILLUSTRATIVE EXAMPLE 1101

1	2	3	4	5	6	7	8
Outflow o	Storage S	$\frac{1}{2}o \cdot \Delta t$	$S - \frac{1}{2}o \cdot \Delta t$	$S + \frac{1}{2}o \cdot \Delta t$	Time to Middle of Step	Inflow at Middle of Step	Outflow at End of Step
c.f.s.	*d.s.f.*	*d.s.f.*	*d.s.f.*	*d.s.f.*	*Days*	*c.f.s.*	*c.f.s.*
20,000	800	10,000	−9,200	+10,800	0	1,000	1,000
30,000	3,300	15,000	−11,700	18,300	1	20,000	31,000
40,000	11,900	20,000	−8,100	31,900	2	105,000	54,500
50,000	38,300	25,000	+13,300	63,300	3	200,000	64,700
55,000	70,700	27,500	43,200	98,200	4	265,000	71,200
60,000	125,100	30,000	95,100	155,100	5	305,000	76,100
65,000	214,200	32,500	181,700	246,700	6	315,000	79,600
70,000	356,300	35,000	321,300	391,300	7	290,000	82,200
75,000	570,600	37,500	533,100	608,100	8	225,000	83,800
80,000	897,400	40,000	857,400	937,400	9	150,000	90,000
84,000	1,230,000	42,000	1,188,000	1,272,000	10	85,000	89,000
85,000	1,243,200	42,500	1,200,700	1,285,700	11	40,000	84,000
87,000	1,260,400	43,500	1,216,900	1,303,900	12	10,000	83,100
90,000	1,279,000	45,000	1,234,000	1,324,000	13	5,000	82,200
100,000	1,327,700	50,000	1,277,700	1,377,700	14	2,000	81,200

the reading on the inflow scale of the rule of Fig. 1102 should equal the reading of the outflow scales at the point where the two equal values are lined up. The reading on the inflow scale of the rule of Fig. 1101 should equal the outflow corresponding to the lined-up values of elevation. This property of the rule could be used to lessen the work of plotting. It seems better, however, to plot both scales and then use it as a check.

PROBLEM

1101. In estimating the cost of a fixed-outlet flood control reservoir, it is necessary to determine the outflow from the reservoir for a flood having the inflow hydrograph shown in the table below. The area-elevation data for the reservoir site are also shown in the table. The discharge from the outlet conduit may be represented by the equation $o = 500E^{1/2}$ acre-feet per hour. Construct a slide rule and route the flood through the reservoir. Use 4 hours as the length of step. In plotting the scales, let 1 inch equal 10,000 acre-feet. This will require two pieces of " parting stop " each 2 feet long. Select clear, straight pieces.

Elevation above Center of Outlet Conduit, in Feet	Surface Area in Acres	Time, in Hours	Inflow, in Acre-Feet per Hour
10	150	0	100
20	250	4	3,000
30	450	8	7,000
40	750	12	11,200
50	950	16	13,600
60	1,250	20	13,500
70	1,600	24	12,000
80	2,000	28	9,000
90	2,400	32	5,600
100	2,800	36	3,500
110	3,200	40	1,700
120	3,600	44	700
		48	100
		(continues constant)	

Level-pool routing with variable stage-discharge relationship. The routing problem becomes more complicated when the discharge through the outlets is not a single-valued function of the stage in the reservoir. The discharge frequently depends upon other factors, such as draft for waterpower, number of outlet gates open, or decrease in discharge capacity of the outlets from submergence by backwater due to a downstream tributary. Many problems of this type can be set up for direct solution by adaptation of the slide-rule method.

If water is withdrawn from the reservoir for irrigation or the generation of power, it may be withdrawn (1) at a constant rate, (2) at a predictable variable rate, or (3) at an unpredictable variable rate. In the first case, the discharge is still a single-valued function of the reservoir stage, and the slide-rule method of the previous section may be used without modification. If the amount of water withdrawn follows some fixed pattern, such as full capacity during times of peak load and half or three-quarters capacity at other times depending upon whether the reservoir stage is above or below a certain elevation, a direct-reading slide rule may be constructed.

Let the discharge rate for power use be represented by Q. Then the equation of continuity for a short step Δt becomes

$$i \cdot \Delta t = \frac{o_1 + o_2}{2} \cdot \Delta t + Q \cdot \Delta t + \Delta S$$

with ΔS positive if the storage is increasing. The rate of discharge through the natural outlets is represented by o, as before, with the sub-

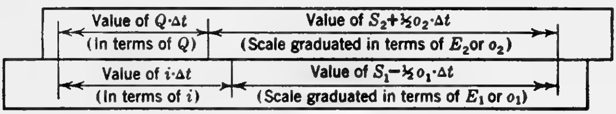


FIG. 1103. Slide Rule for Routing Inflow through a Reservoir when Part of the Water is Discharged for Power, According to a Known Pattern, and Part Discharges Through Fixed Outlets.

scripts 1 and 2 referring to rates at the beginning and end of the step. Transposing,

$$i \cdot \Delta t + (S_1 - \tfrac{1}{2} o_1 \cdot \Delta t) = (S_2 + \tfrac{1}{2} o_2 \cdot \Delta t) + Q \cdot \Delta t \qquad [1109]$$

The method of graduating the scales for direct solution of this equation is shown in Fig. 1103.

A less simple case occurs when the discharge from the fixed outlets is affected by submergence, as from tidal waters or from backwater due to a flood in a large downstream tributary. In order for the problem to be solved, the value of the discharge for every possible combination of values of reservoir stage and tributary flow must be known, and the hydrograph of the tributary must be known, as well as the hydrograph of the inflow into the reservoir. Let the flow in the tributary be represented by F. Then, as before

$$i \cdot \Delta t + (S_1 - \tfrac{1}{2} o_1 \cdot \Delta t) = (S_2 + \tfrac{1}{2} o_2 \cdot \Delta t)$$

but now $(S_1 - \frac{1}{2}o_1 \cdot \Delta t)$ and $(S_2 + \frac{1}{2}o_2 \cdot \Delta t)$ are functions of F and E instead of E only. A direct solution may be obtained by use of the sliding charts illustrated in Fig. 1104.

The index at the left end of the upper slide is set in line with the inflow for the step, read on the scale at the left of the lower slide. The point corresponding to the value of the stage and tributary flow at the beginning of the step is located on the lower slide. Assume that this point is the point A shown in the figure. If the tributary flow does not change

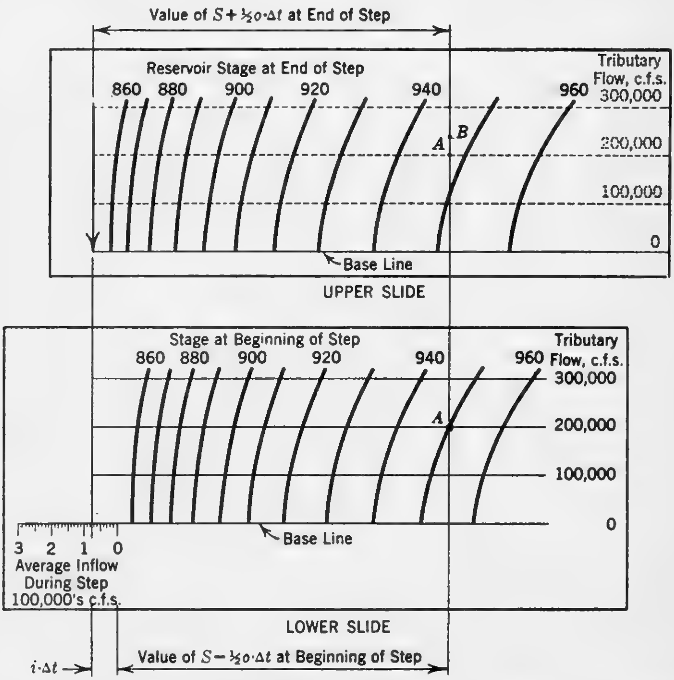


FIG. 1104. Slide Chart for Flood Routing When Discharge from Reservoir is Affected by Backwater from a Downstream Tributary.

The upper slide is drawn in black on thin celluloid, and the lower slide in red on white board. The two parts are then fixed so that they will slide with their base lines in coincidence. The dotted lines and figures on the upper slide may be omitted since their position will be directly above the same lines and figures on the lower scale.

appreciably during the step, the stage at the end of the step may be read on the upper slide, directly above point A. If the tributary flow does change appreciably during the step, the stage at the end of the step is determined approximately by the location of point B, in line with A, but at the correct value of the tributary flow at the end of the step. This procedure should not be depended upon to give good results if the

tributary flow changes very appreciably during the step. The remedy is to make a new chart based upon a shorter length of step Δt.

The submergence effect due to downstream backwater is not the only effect that can be included by the method shown in Fig. 1104. Other independent variables that affect the discharge-stage relation, such as the number of gates open, may be treated in the same way.

Storage under the backwater curve: superstorage. For extremely long reservoirs in the valleys of rivers having mild slope, the assumption of a level pool may be seriously in error, for the volume of water between a horizontal plane through the water surface at the lower end of the pool, and the actual water surface, which follows a backwater curve, may be considerable. This part of the storage may be referred to as superstorage. If the river valley is essentially uniform the volume of superstorage is a function of the inflow, and may be evaluated by computing backwater curves for various inflows in a typical section of the valley. For valleys that are not uniform, the volume of superstorage is a function of both the inflow and the stage in the reservoir. A large amount of data and many computations would have to be made before this functional relationship could be summarized, but its effect can be included in the routing, if deemed necessary, by a method similar to that of the previous article. We assume that the stage-discharge relationship is fixed, and not affected by any other variables. Therefore equation (1108) applies, but $(S_1 - \frac{1}{2}o_1 \cdot \Delta t)$ and $(S_2 + \frac{1}{2}o_2 \cdot \Delta t)$ are functions of the inflow as well as of the reservoir stage. The slide chart with the transparent upper slide is drawn in exactly the same manner as that of Fig. 1104, but the horizontal lines labeled " tributary flow " are now labeled " inflow." This makes two " inflow " scales on the same chart. The left-hand scale represents the average inflow during the step, while the right-hand scale represents the inflow at the beginning or end of the step. The rule must be read with point B in line with point A, but not coincident with it unless the inflow remains constant during the step. This slide-rule solution may be used whether the superstorage is or is not a function of the stage. In the former case the collection, computation, and tabulation of the data is a formidable task.

In selecting the length of step to be used in the routing computations, two requirements must be complied with. First, the step must be short enough so that the shape of inflow hydrograph is reproduced with satisfactory accuracy. Second, the step must be long compared with the time necessary for a wave to travel from the upper end of the pool to the lower end. If both of the requirements cannot be met, the problem is not of the type of slowly varied flow that can be treated as steady flow.

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